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A PRINCIPLE OF EQUIVALENCE OF MODALITIES DE-DICTO AND DE-RE UNDER THE CONDITION OF A-PRIORI-NESS OF KNOWLEDGE, FORMULATED BY AN ARTIFICIAL LANGUAGE OF FORMAL MULTIMODAL AXIOMATIC THEORY Φ^{DR}

The *subject-matter* of investigation is the nontrivial modal-logic problem of equivalence of *de-dicto* and *de-re* types of modalities. The *target* – explicating, exact formulating and adding the precisely formulated principle of equivalence of the modality-types as a new proper philosophy axiom scheme to the multimodal axiomatic system of formal philosophy Φ+∃. The *scientific novelty*: for realizing the target, the already published formal axiomatic theory Φ+∃ has been transformed into a qualitatively different logically formalized multimodal axiomatic system called Φ^{DR} due to manifest taking into an account and systematical dealing with the different modality types called *de-dicto* and *de-re*. For accomplishing the transformation, (1) definitions of the alphabets of artificial object-language and meta-language of Φ+∃ have been changed: new symbols Ω^{D} and Ω^{R} standing for perfection-modalities (belonging to the types D and R, respectively) have been included into the alphabet of metalanguage of Φ^{DR}; (2) in Φ^{DR}, one new axiom-scheme (containing the new symbols Ω^{D} and Ω^{R}) has been added to the set of axiom-schemes of Φ+∃; (3) by means of the artificial language of Φ^{DR}, a precise formulation of the principle of equivalence of the modality-types *de-dicto* and *de-re* is given, and the concrete epistemic condition, under which the equivalence principle is valid, is defined. For the first time, a nontrivial *multimodal* interpretation is given for the traditional formal logic square of opposition and hexagon. Also, for the first time, these geometric figures have been utilized for visual modeling a system of *formal-axiological* relations among qualitatively different *de-re*-perfection-modalities as *evaluation-functions*.

Keywords: modalities; de-dicto; de-re; principle-of-equivalence-of-modalities-de-dicto-and-de-re; logically-formalized-multimodal-axiomatic-philosophy-system.

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ПРИНЦИП ЭКВИВАЛЕНТНОСТИ МОДАЛЬНОСТЕЙ ДЕ-ДИКТО И ДЕ-РЕ ПРИ УСЛОВИИ АПРИОРНОСТИ ЗНАНИЯ, СФОРМУЛИРОВАННЫЙ НА ИСКУССТВЕННОМ ЯЗЫКЕ ФОРМАЛЬНОЙ МУЛЬТИМОДАЛЬНОЙ АКСИОМАТИЧЕСКОЙ ТЕОРИИ $Ф^{DR}$

Предмет исследования – нетривиальная модально-логическая проблема эквивалентности *de-dicto* и *de-re* типов модальностей. *Цель* – прояснение, точная формулировка, и добавление уточненного принципа эквивалентности этих типов модальностей в качестве новой схемы собственных аксиом в мультимодальную аксиоматическую систему формальной философии Ф+∃. *Научная новизна*: *для* достижения этой цели, уже опубликованная формальная аксиоматическая теория Ф+∃ преобразована в некую качественно отличную от нее логически формализованную мультимодальную аксиоматическую систему, названную $Ф^{DR}$, благодаря принятию во внимание и систематическому использованию в ней упомянутых различных типов модальностей, именуемых *dicto* и *de-re*. Для осуществления такой трансформации, (1) изменены определения алфавитов языка-объекта и метаязыка теории Ф+∃: в алфавит метаязыка теории $Ф^{DR}$ добавлены новые символы Ω^{D} и Ω^{R}, обозначающие некие (любые) модальности совершенства упомянутых типов D и R, соответственно; (2) ко множеству аксиомных схем теории Ф+∃, в теорию $Ф^{DR}$ добавлена одна новая схема аксиом, содержащая символы Ω^{D} и Ω^{R}; (3) на искусственном языке $Ф^{DR}$ дана точная формулировка принципа эквивалентности типов модальностей *dicto* и *de-re*, а также определено то конкретное эпистемическое условие, при котором этот принцип эквивалентности имеет законную силу. Впервые предложена некая нетривиальная *мультимодальная* интерпретация традиционного формально-логического квадрата и гексагона логической оппозиции абстрактных понятий. Также впервые упомянутые геометрические фигуры использованы в качестве наглядных моделей системы *формально-аксиологических* отношений между качественно различными *de-re*-модальностями-совершенства как *ценностными функциями*.

Ключевые слова: модальности; de-dicto; de-re; принцип эквивалентности модальностей de-dicto и de-re; логически формализованная мультимодальная аксиоматическая система философии.

1. Introduction

Many philosophers have defended an account of *de re* belief about an object in terms of having some *de dicto* belief about that object while also bearing a relation of acquaintance to it, that is, while being epistemically *en rapport* with the object [Wierenga 2021].

The most able and influential enemy of modality (both *de dicto* and *de re*) was W. V. Quine, who vigorously defended both the following theses. First, that modality *de dicto* can be understood only in terms of the concept of analyticity (a problematical concept in his view). Secondly, that modality *de re* cannot be understood in terms of analyticity and therefore cannot be understood at all [Inwagen, Sullivan, and Bernstein 2023].

De re is less favorable than *de dicto* [Zhang and Davidson 2020, p. 1].

Formal-logic aspect of alethic modalities was noticed and systematically studied yet in ancient times, at least, since that famous epoch when the syllogism theory had been created and elaborated in details [Аристотель 1978; Лукасевич 1959; Маркин 2018]. However, at first, the nontrivial distinction between *de re* and *de dicto* types of the modalities had been not-well-recognized.

> Aristotle considered some "mixed" syllogisms with one apodeictic premise, one assertoric premise and apodeictic conclusion to be valid. His pupils Theophrastus and Eudemus introduced the principle that the conclusion always has the same modal character as the weaker of the premises, thereby they rejected all mixed modal syllogisms. In medieval logic, a distinction was made between de dicto and de re modalities. It was demonstrated that propositions with de dicto and de re modalities have different deductive characteristics. Aristotle's apodeictic syllogistic contains both: reasonings valid only under de dicto-interpretation of modalities … and reasonings valid only under de re-interpretation (e.g. modus Barbara). When we accept the "principle of the weakest premise", apodeictic syllogistic can be naturally interpreted as containing de dicto modalities. The eminent Polish logician Jan Lukasiewicz suggested that both modal syllogistic versions were incorrect. In his opinion all mixed modi formed from the valid categorical syllogisms (e.g. Barbara rejected by Aristotle) are also valid. Lukasiewicz justified these modi

by means of his positive assertoric syllogistic and four-valued modal logic, which contains some theorems unprovable in normal modal calculi [Маркин 2018, c.114].

In Middle Ages, when the distinction between *de re* and *de dicto* types of the modalities had been well-recognized and established as a norm (custom) of intellectually respectable theorizing in logic, systematical discussing knotty interrelations between the two well-separated types of modalities was progressively developed with respect to main themes of philosophizing of that time [Knuuttila 2021; Lagerlund 2022; Parsons 2014].

In contemporary literature accepting and following the medieval distinction of *de dicto* and *de re*, profound investigations of the two types of modalities can be found, for example, in [Prior 1952; Quine 1956; Kneale 1962; Kaplan 1968; Sosa 1970; Chisholm 1976; Burge 1977; Целищев 1978; Lewis 1979; McKay 1991; Reimer 1995; Salmon 1997; Jeshion 2002; Stanley 2002; Taylor 2002; Хлебалин 2003; 2004; Turner 2010; Keshet 2010; Roca-Royes 2011; Ламберов 2011; Keshet, and Schwarz 2014; Куайн 2010; 2016; Маркин 2018; Wierenga 2021; Inwagen, Sullivan, and Bernstein 2023; Nelson 2019; 2023; Schwitzgebel 2024] and in many other noteworthy writings on modal logic, metaphysics, and philosophy of logic. Summarizing results of contemporary investigations of the relationship between modality types *de dicto* and *de re*, one is to affirm that *there is no logic equivalence between the two* [Маркин 2018, c.109]. Consequently, it is quite natural to anticipate not the equivalence (interchangeability) of the two, but its negation.

Hence, with respect to the habitual principle of logic separation of *de dicto* and *de re*, the manifestly challenging (surprising and puzzling) title of present article is psychologically unexpected (at least queer; requiring explanations). Therefore, at the very beginning, the introduction to this article must clarify the ambiguous meaning of the title. One of the possible meanings (namely, the implied at first) is certainly false, but there is another meaning (qualitatively different one) to be defined precisely by means of the hitherto never published multimodal axiomatic theory Φ^{DR}.

First necessary step on the way to adequate understanding the paper title (destroying the *illusion* of its paradoxality, countedintuitiveness) is attracting attention to the phrase "under the *condition* of *a-priori-ness* of knowledge". *Unconditional* affirming the equivalence does make up a shocking paradox, but in the title under discussion, *the equivalence is affirmed not unconditionally*, but under a *very special* (even *extraordinary*) *condition* concerning knowledge. What does this condition mean? How and by which means can one decide whearther a knowledge is *a priori* or not? The answer is given by means of the mentioned multimodal formal axiomatic theory Φ^{DR} to be precisely defined below in this article. Certainly, this is not a direct answer, but it is quite sufficient for the introduction. More comprehensible and developed answer is to be given by the present article *as a whole*.

However, already in the introduction, it is worth providing at least some informal tips, vaguely formulated intuitive guesses concerning the main direction in which the discourse is to move in this paper. Which concrete facts and abstract conceptual materials are to be taken into an account and kept in mind while approximating to the promised precise axiomatic definition of the puzzling principle of equivalence of *de re* and *de dicto* types of modalities?

To answer this quite reasonable question, it is worth taking into an account the curious fact (influential tendency started since the beginning of XX century) in history of philosophy of modal logic, namely; (1) there is a propensity to locate *de dicto* modalities into the realm of logic-and-sciences proper: (2) there is a propensity to locate *de re* modalities into the realm of proper metaphysics (proper philosophical ontology), axiology, and even theology [Inwagen, Sullivan, and Bernstein 2023].

Modality *d dicto* is the modality of propositions ('dictum' means proposition, or close enough). If modality were coextensive with modality *de dicto,* it would be at least a defensible position that the topic of modality belongs to logic rather than to metaphysics. (Indeed, the study of modal logics goes back to Aristotle's Prior Analytics.)

But many philosophers also think there is a second kind of modality, modality *de re – modality of things. (The* modality of substances, certainly, and perhaps of things in other ontological categories.) The status of modality *de re* is undeniably a metaphysical topic, and we assign it to the "new" metaphysics because, although one can ask modal questions about things that do not change – God, for example, or universals – a large proportion of the work that has been done in this area concerns the modal features of changing things [Inwagen, Sullivan, and Bernstein 2023].

Thus, there is a curious (even somewhat elegant) *analogy* between the ordered couples <metaphysics-proper; positive-sciences-proper> and <*de-re*-modalities; *de-dicto*-modalities>. Although, in relation to the logic theory of quantification, Quine has a negative attitude to both *de re* and *de dicto* modalities, while reading his writings on the theme, it easy to feel that his attitude to *de re* is much worse than to *de dicto*: his rejecting *de re* is analogous to his rejecting metaphysics: both are meaningless, from his point of view [Куайн 2010, с. 200-228; 2016, с. 150-177]. In this concrete relation, Quine has been not alone: many positivist-minded logicians have believed that *de re* interpretation of modalities is a manifestation of *essentialism* which (according to them) is a kind of metaphysics in the positivist meaning of the word [Хлебалин 2003, с. 36, 39, 41]. It is highly likely that, at least at the level of subconsciousness, just the anti-metaphysics is underlying Quine's resolutely negative attitude to combining the standard logic of quantification with the logic of modalities. If one (for example [Williamson 2013]) is right, when the one considers *modal logic as metaphysics*, then it is highly likely (and quite natural to anticipate) that radical positivists are to reject if not all then at least some results of combining quantifiers with modalities, especially with *de re* ones [Куайн 2010, с. 213, 215-217, 219-223; 2016, с. 168, 172,173]. It is not merely curious but also quite relevant to note here that the fact-fixing statement "*de re* is less favorable" has been established (demonstrated statistically) by an *empirical* investigation of belief reports [Zhang and Davidson 2020].

The tendency completely to eliminate (exclude) *de re* type of modalities from meaningful discourse is *analogous* to the positivist tendency to eliminate metaphysics, axiology and theology from scientific philosophy. The above-indicated facts *induce* a psychologically unexpected *hypothesis* that, probably, there is a hidden (not quite evident) but essential link between mutually corresponding formal-logical systems of *de re* modalities and *formal-axiological* systems of *evaluation-functions* making up the subject-matter of *metaphysics-proper* understood (interpreted) as nothing but abstract *formal axiology* (such an unhabitual hypothesis is formulated and investigated, for instance, in [Лобовиков 2007; Lobovikov 2022]). Obviously, from the traditional point of view, this is a somewhat strange guess. However, in my opinion, this hypothesis is worth taking into an account and investigating carefully by the hypothetic-deductive method (I intend to do this in the present article).

As the long and hard discussion about the significant difference between modality-types *de dicto* and *de re* still goes on, I have decided to submit this paper formulating quite a new idea concerning the problem. In the present paper, the new nontrivial idea (unexpected and somewhat puzzling one) concerning modalities *de dicto* and *de re* is represented by a new scheme of proper axioms of formal philosophy (namely, AX-7) making up the novel formal multimodal axiomatic theory Φ^{DR}.

2. Methods

In the present article, the hypothetic-deductive method and discrete mathematical modeling are exploited. Intentional constructing artificial languages and formal axiomatic theories possessing special properties is undertaken systematically. The artificial languages are used for precise defining formal axiomatic theories modeling multimodal proper philosophy systems. For obtaining qualitatively new (hitherto never published) nontrivial scientific results, some significant mutations (creating new species) are made in some already existing (previously created) artificial languages and formal axiomatic theories. For example, alphabets of object-languages are enriched, definitions of terms and formulae are changed, some (at least one) *new axiom-scheme is added* to the set of proper-axiom schemes. Owing to such significant mutations creating new species of formal axiomatic theories, qualitatively new possibilities of formalization and interpretation appear and new application domains are discovered.

New Scientific Results

1.1. A New Logically Formalized Multimodal Axiomatic Philosophy System Φ^{DR}

The hitherto never considered multimodal formal axiomatic theory Φ^{DR} is a result of significant mutation of the formal axiomatic *epistemology-and-axiology-and-ontology* theory Φ+∃ [Lobovikov, 2024).

For exact defining the formal theory Φ^{DR}, it is necessary to provide exact definitions of the notions: "*alphabet* of object-language of Φ^{DR}"; "*term* of Φ^{DR}"; "*formula* of Φ^{DR}"; "*axiom* of Φ^{DR}". Exact definitions of these notions of Φ^{DR} are strikingly *analogous* (look *similar*) to the exact definitions of the corresponding notions of Φ+∃, which definitions are already utilized and published in [Lobovikov, 2024]. Therefore, one can feel *déjà vu* (strong sensation of *analogousness*) and fall into the *identity illusion*. Nevertheless, for the sake of rigor, in the present article, it is necessary manifestly to give precise definitions of notions: "*alphabet* of object-language of Φ^{DR}"; "*term* of Φ^{DR}"; "*formula* of Φ^{DR}"; "*axiom* of Φ^{DR}", in spite of the mentioned *analogousness*, as the words *"analogousness" and "identity" are not synonyms.* Meanings of these words are not logically equivalent. Notions of Φ^{DR} *differ substantially* from the corresponding *analogous (similar)* notions of Φ+∃. Therefore, it is reasonable to start with precise formulating definitions of those notions which are indispensable for adequate comprehending this paper, in spite of the false sensation that they are just repetitions of the already published statements. Let us begin with precise defining the notion "object-language-alphabet of Φ^{DR}".

According to the below-provided definition (containing 11 items), the alphabet of object-language of Φ^{DR} includes all the signs which belong to the alphabets of object-languages of Σ, Σ+C, Σ+2C, and Φ+∃. But, the conversion of this judgement is not true, because, in Φ^{DR}, some new signs are added to the alphabets of object-languages of Σ, Σ+C, Σ+2C, and Φ+∃. The result of such a significant change (complementation) is the below-placed exact definition of the alphabet of object-language of Φ^{DR}.

1) The lowercase Latin letters p, q, d (and the same letters possessing lower number indexes) are elements of the object-language-alphabet of Φ^{DR}. Such and only such lowercase Latin letters are called

"*dictum variables*". In the object-language-alphabet of Φ+∃, *not all lowercase Latin letters are called dictum variables* because, according to the provided definition, such lowercase Latin letters which are elements of the set {g, b, e, n, x, y, z, s, h, a, t, f} do not belong to the set of *dictum variables* of object-language of Φ^{DR}.

2) The lowercase Latin letters s, h, a (and the same letters having lower literal indexes: s_m, h_s, a_t,) are elements of the object-language-alphabet of Φ^{DR}. The mentioned lowercase Latin letters (and only they) are called "*dictum constants*".

3) The habitual proper (pure) logic symbols $\supset, \neg, \leftrightarrow, \&, \vee$ called, respectively, "classical (or 'material') implication", "classical negation", "classical equivalence", "classical conjunction", "classical non-excluding disjunction" are elements of the object-language-alphabet of Φ^{DR}.

4) Elements of the set {I^D, $\square^D$, Ж^D, Щ^D, H^D, K^D, A^D, E^D, S^D, Z^D, G^D, W^D, O^D, B^D, C^D, Y^D, T^D, F^D, P^D, ©^{D}, D^D, U^D, J^D}, containing the signs $\square^D$, ©^{D}, the capital Cyrillic letters Ж and Щ having upper literal index D, and some (but not all) capital Latin letters possessing no number indexes but having upper literal index D (indicating to "de dicto"), belong to the object-language-alphabet of Φ^{DR}. The mentioned elements of the alphabet are called "de-dicto-modality-symbols" in Φ^{DR}. The de-dicto-modality symbol Ж^D belongs *only* to the object-language-alphabets of Φ+∃ and Φ^{DR}. The de-dicto-modality symbols I^D, Щ^D, H^D, Z^D, and ©^{D} belong *exclusively* to the object-language-alphabet of Φ^{DR}.

5) Elements of the set {I^R, $\square^R$, Ж^R, Щ^R, H^R, K^R, A^R, E^R, S^R, Z^R, G^R, W^R, O^R, B^R, C^R, Y^R, T^R, F^R, P^R, ©^{R}, D^R, U^R, J^R}, containing the signs $\square^R$, ©^{R}, the capital Cyrillic letters Ж and Щ having upper literal index R, and some (but not all) capital Latin letters possessing no number indexes but having upper literal index R (indicating to "de re"), belong to the object-language-alphabet of Φ^{DR}. The mentioned elements of the alphabet are called "de-re-modality-symbols" in Φ^{DR}. The de-re-modality symbols Ж^R, I^R, Щ^R, H^R, Z^R, and ©^{R} belong *exclusively* to the object-language-alphabet of Φ^{DR}.

6) The lowercase Latin letters *z*, *y*, *x* (and the same letters having lower number indexes) belong to the object-language-alphabet of Φ^{DR}. They are called "*axiological variables*" in Φ^{DR}.

7) Also the lowercase Latin letters “b” and “g” belong to the object-language-alphabet of Φ^{DR}. They are called “*axiological constants*”.

8) The capital Latin letters C^1, K^1, K^2, E^1, E^2, C^2, $A_k{}^n$, $C_j{}^n$, $B_i{}^n$, $D_m{}^n$, …, having number-indexes, are elements of the alphabet of object-language of Φ^{DR}. These capital Latin letters are called “*axiological-value-functional-symbols*”. The upper number-index *n* (in these symbols) means that the axiological-value-functional-symbol (indexed by number *n*) is *n*-placed one. Generally speaking, axiological-value-functional-symbols may possess no lower number-index. However, if axiological-value-functional-symbols possess lower number-indexes, then, if these indexes are different, then the indexed axiological-value-functional-symbols are different signs.

9) The “round brackets”, namely, “(” and “)” belong to the object-language-alphabet of Φ^{DR}. These auxiliary symbols are utilized in the present article as usually in symbolic logic, namely, as pure technical signs.

10) The “square brackets”, namely, “[” and “]” belong to the object-language-alphabet of Φ^{DR}. But, it is very important to emphasize here that the “square brackets” (in contrast to the “round ones”,) are exploited in Φ^{DR} not as the auxiliary (pure technical) signs, but as *ontologically meaningful* symbols. Being a psychological surprise, such unhabitual (odd) using the “square brackets” makes some psychological difficulty, because, for common people using natural language in everyday life, square brackets and round ones seem identical and, as a rule (statistic norm) the two kinds of brackets are exploited (in natural language of common people) as synonyms. However, in contrast to natural language of common people, in the artificial object-language of Φ^{DR}, the two species of brackets possess *qualitatively different* meanings. They play *substantially different* roles in Φ^{DR}: round brackets are used as purely technical (auxiliary) signs; on the contrary, usage of square brackets has an *ontological* meaning (very important one). The *ontological* meaning of the inhabitual (even odd) usage of square brackets in the artificial object-language of Φ^{DR}, is exactly defined below in that special part of the present article which (part) is devoted to exact defining *semantics* of object-language of Φ^{DR}. However, even

at the purely syntactic level of the object-language of Φ^{DR}, square-bracketing plays a significant role in giving precise definition of/for the concept "formula of Φ^{DR}". (This precise definition is to be given below in the present section of the given article.) Moreover, the unusual utilization of square brackets *plays an important role* also in the precise formulations of some schemes of axioms of Φ^{DR}." (These formulations of axiom-schemes are placed below also in the present section of the given article.)

11) An inhabitual (odd) complex symbol "=+=" (composed of the habitual symbols) belongs to the object-language-alphabet of Φ^{DR}. The symbol "=+=" is called (a sign of) "*formal-axiological equivalence*". The strange compound sign "=+=" *plays a very important role* in the precise definition of the notion "formula of Φ^{DR}" and also in the precise formulations of some schemes of axioms of Φ^{DR}.

12) A symbol belongs to the object-language-alphabet of Φ^{DR}, when and only when the symbol is an element of that alphabet due to the above-formulated points 1) – 11) of this definition.

Any finite queue (succession) of signs is called "an *expression* of Φ^{DR}", when and only when that queue contains such and only such signs which belong to the object-language-alphabet of Φ^{DR}.

A precise definition of the concept "*term* of Φ^{DR}" is the following.

1) the *axiological variables* (mentioned in the above-given definition of object-language-alphabet of Φ^{DR}) are terms of Φ^{DR}.

2) the *axiological constants* (mentioned in the above-given definition of object-language-alphabet of Φ^{DR}) are terms of Φ^{DR}.

3) the *dictum variables* (mentioned in the above-provided definition of object-language-alphabet of Φ^{DR}) are terms of Φ^{DR}.

4) the *dictum constants* (mentioned in the above-provided definition of object-language-alphabet of Φ^{DR}) are terms of Φ^{DR}.

5) If Φ_k^n is an *n-placed value-functional symbol* (belonging to the above-provided definition of object-language-alphabet of Φ^{DR}), and $t_i, \dots t_n$ are *terms* of Φ^{DR}, then any expression having the form $\Phi_k^n t_i, \dots t_n$ is a term of Φ^{DR}. (Here, it is worth keeping in mind that symbols $t_i, \dots t_n$ belong not to the object-language but to the meta-language as they stand for *any* terms of Φ^{DR}; the similar note is worth keeping in mind with respect to the symbol Φ_k^n also belonging to the meta-language of Φ^{DR}.)

6) When t_n is a term of Φ^{DR}, and the sign Ψ (belonging to the meta-language of Φ^{DR}) stands for a (any) modal-symbol belonging to the set {I^R, $□^R$, $Ж^R$, H^R, K^R, A^R, E^R, S^R, Z^R, T^R, F^R, P^R, $©^R$, D^R, C^R, Y^R, G^R, W^R, O^R, B^R, U^R, J^R}, then all such object-language expressions of Φ^{DR}, which (expressions) have the form Ψt_n, are terms of Φ^{DR}. Here, it is worth taking into an account, that, strictly speaking, the expression possessing form Ψt_n is not a term of Φ^{DR}, but a scheme of terms of Φ^{DR}. (Thus, de-re modalities applied to terms make up new terms.)

7) Any expression of the object-language of Φ^{DR}, is a term of Φ^{DR}, then and only then, when this is so due to the points 1) – 6) of the given definition.

Now, the *pure-syntax* aspect of the abstract notion "*term* of Φ^{DR}" is perfectly defined. Hence, now it is appropriate to move on to exact definining the *pure-syntax* aspect of the abstract notion "*formula* of Φ^{DR}". To accomplish this move correctly, let us make an agreement that in the given article, lowercase Greek letters ω, β, and α (belonging to meta-language of Φ^{DR}) stand for *any* formulae of Φ^{DR}. Taking this agreement into an account, it is possible to formulate the below-placed precise definition of the notion "formula of Φ^{DR}".

1) All such lowercase Latin letters which are named "*dictum variables*", and also all such lowercase Latin letters which are named "*dictum constants*", are elements of the set of formulae of Φ^{DR}.

2) When α and β are formulae of Φ^{DR}, then all such object-language expressions of Φ^{DR}, which (expressions) have the forms $(\alpha \,\&\, \beta)$, $(\alpha \vee \beta)$, $(\alpha \supset \beta)$, $(\alpha \leftrightarrow \beta)$, $\neg\alpha$, are elements of the set of formulae of Φ^{DR} as well.

3) When t_i and t_k are terms of Φ^{DR}, then $(t_i =\!+\!= t_k)$ is a formula of Φ^{DR}.

4) When t_i is a term of Φ^{DR}, then $[t_i]$ is a formula of Φ^{DR}.

5) When α is a formula of Φ^{DR}, and the sign Ψ (belonging to the meta-language of Φ^{DR}) stands for a (any) modal-symbol belonging to the set {I^D, $□^D$, $Ж^D$, $Щ^D$, H^D, K^D, A^D, E^D, S^D, Z^D, T^D, F^D, P^D, $©^D$, D^D, C^D, Y^D, G^D, W^D, O^D, B^D, U^D, J^D}, then all such object-language expressions of Φ^{DR}, which (expressions) have the form $\Psi\alpha$, are formulae of Φ^{DR}. Here, it is worth taking into an account, that, strictly speaking, the expression possessing form $\Psi\alpha$ is not a formula of Φ^{DR}, but

a scheme of formulae of $Ф^{DR}$. Thus, *de-dicto* modalities applied to formulae make up new formulae. This is a significant *syntactic* difference between the modalities *de dicto* and *de re* (in $Ф^{DR}$). The first ones are applied to *formulae* while the second ones are applied to *terms*.

6) Finite successions (queues) of symbols belonging to the object-language-alphabet of $Ф^{DR}$ are formulae of $Ф^{DR}$, if and only if this is so due to the points 1) – 5) of this definition.

This concrete part of the article is intentionally reduced exclusively to *syntactic* meanings of object-language-expressions of $Ф^{DR}$. Therefore, the set of *de-dicto*-modal-symbols {I^D, $\square^D$, $Ж^D$, $Щ^D$, H^D, K^D, A^D, E^D, S^D, Z^D, T^D, F^D, P^D, $©^D$, D^D, C^D, Y^D, G^D, W^D, O^D, B^D, U^D, J^D} and the set of corresponding *de-re*-modal-symbols are considered here as nothing but sets of extremely short *names*. The sign I^D (Latin letter «I» possessing the upper literal index D informing that the modality belongs to the type called "*de dicto*") is a name of/for the *newly* introduced philosophical-*ontology* modality "it is *immutable* (*constant*, does not undergo a change) that …". The sign $\square^D$ is a short name for the well-known *de dicto* modality "it is *necessary* that …". The sign $Ж^D$ (Cyrillic letter Ж possessing the upper literal index D) is a short name of/for the *de dicto* philosophical-*ontology* modality "what is indicated-and-described by the *dictum* …, *exists*". (This unhabitual de-dicto-modality has been introduced originally in [Lobovikov 2024].) The sign $Щ^D$ (Cyrillic letter Щ possessing the upper literal index D) is a short name of/for the *de dicto* philosophical modality "it is *essential* (or it is an *essence*) that …". (Hence, at least some of the nontrivial problems of the essentialism can be modeled in $Ф^{DR}$.)

Here it is worth highlighting that the symbols $Ж^D$ and I^D (denoting the ontological modalities *de dicto*) do not belong to the object-language-alphabets of Σ, Σ+C, Σ+2C, Ф. The formal theory $Ф^{DR}$ is an outcome of adding the symbols I^D, $Щ^D$, H^D, Z^D, and $©^D$ to the set of de-dicto-modality-symbols belonging to the object-language-alphabet of Ф+∃. Also, it is worth highlighting here that, in contrast with the object-language-alphabets of Σ, Σ+C, Σ+2C, Ф, and Ф+∃, the modality signs in $Ф^{DR}$ possess upper literal indexes D or R informing that the indexed symbols stand, respectively, for *de-dicto* or *de-re* modality types.

The symbols H^D, K^D, E^D, A^D, S^D, Z^D, T^D, F^D, P^D, $©^D$, D^D, Y^D, C^D, respectively, denote the following modal expressions: "it is *impersep-tible (sensually unverifyable, inaccessible to the senses)* that…"; "agent *Knows* that…"; "agent *Empirically* (*a-posteriori) knows* that…"; "agent *A-priori knows* that…"; "in some fixed time-and-space, under some special conditions, an agent has a *Sensation* (either by means of some instruments, or immediately), that…"; "in some fixed time-and-space, under some special conditions, (either immediately, or by means of some instruments) an agent *changes* what is described by the dictum that …". "it is *True* that…"; "agent has *Faith* that… (or person believes that…)"; "in a consistent theory, it is *Provable* that…"; "it is *Constructive* that…"; "it is *Decidable* that…"; "it is *Complete* that …"; "it is *Consistent* that…".

The symbols G^D, W^D, O^D, B^D, U^D, J^D, respectively, denote the following modal expressions: "it is *Good* (perfect in moral sense) that…"; "it is *Wicked* (bad in moral sense) that…"; "it is *Obligatory* (or it is a duty) that …"; "it is *Beautiful* (perfect in aesthetic sense) that …"; "it is *Useful* (beneficial) that …"; "it is a *Joy* (pleasure, gladness, happiness) that …". In this concrete part of the article, purely *syntactic* meanings of the de-dicto-modality-symbols are precisely defined by the below-placed schemes of proper-formal-philosophy-axioms of the multimodal system Φ^{DR}. Obviously, the axiomatic definition is not manifest (direct) one, but, nevertheless, it is quite exact and sufficient for precise philosophizing.

In the logically formalized multimodal philosophy system Φ^{DR}, the proper axioms of universal philosophical *epistemology*, formal philosophical *ontology*, and abstract formal *axiology* are combined with pure logic axioms which (axioms of logic-proper) are obviously *analogous* to the axioms of classical propositional logic (let it be called PL). Thus, pure-logic axioms and logical derivation rules of Φ^{DR} are substantially *analogous* to the ones of classical PL calculus. Certainly, the analogousness is not an absolute identity as sets of formulae of PL and Φ^{DR} are different.

The well-known proper logic meanings of the classical propositional-logic-connectives represented by habitual signs $\neg$, &, $\leftrightarrow$, $\vee$, and $\supset$, which (signs) are exploited in the below-provided axiom-schemes of Φ^{DR}, are precisely defined, for instance, in [Kleene 1957;

1973]. Schemes of axioms and logic-derivation rules of classical PL calculus are also well-defined in [Kleene 1957; 1973]. Therefore, let us abstain from repetition of the well-known, and concentrate on actually original aspects of Φ^{DR}.

While comparing formal theories $\Phi+\exists$ and Φ^{DR}, one can come to the *truthlike* conclusion that corresponding definitions of their basic notions are completely identical. But, strictly speaking, being actually *analogous* they are not identical. A*nalogousness* is *conjunction* of *similarity* and *difference*. (This very important statement is well-demonstrated visually by the relevant hexagon and square of opposition modeling graphically the logical relations among notions “difference”, “identity”, “similarity”, and “analogousness” [Beziau 2015].) Being *similar*, $\Phi+\exists$ and Φ^{DR} have *different* alphabets of their object-languages and, hence, *different* sets of artificial-language expressions, *different* sets of terms (and, hence, *different* sets of formulae), different sets of proper axioms, and, hence, *different* sets of theorems. Thus, even at the syntaxis level, $\Phi+\exists$ and Φ^{DR} are qualitatively *different* axiomatic theories.

In the present section of the paper, the *syntactic* meanings of the modality signs and of the other symbols belonging to the alphabet of object-language of Φ^{DR} are defined precisely by the below-located list (AX-1 – AX-11) of schemes of proper-philosophy (ontology, epistemology, axiology) axioms of Φ^{DR}. (Obviously, the *axiomatic* definition of proper-philosophy notions, namely, universal epistemological, ontological and axiological concepts is *not a manifest* definition. Notwithstanding, it is *perfectly exact* one.) When α, β, ω are formulae of Φ^{DR}, then all such (and only such) expressions of the object-language of Φ^{DR}, which have the following logic forms, are *proper-axioms* of Φ^{DR}.

Axiom scheme AX-1: $A^D\alpha \supset (\Omega^D\beta \supset \beta)$.

Axiom scheme AX-2: $A^D\alpha \supset (\Omega^D(\alpha \supset \beta) \supset (\Omega^D\alpha \supset \Omega^D\beta))$.

Axiom scheme AX-3:

$A^D\alpha \leftrightarrow (K^D\alpha \;\&\; (\neg\Diamond^D\neg\alpha \;\&\; \neg\Diamond^D S\alpha \;\&\; \Box^D(\beta \leftrightarrow \Omega^D\beta)))$.

Axiom scheme AX-4:

$E^D\alpha \leftrightarrow (K^D\alpha \;\&\; (\Diamond^D\neg\alpha \vee \Diamond^D S\alpha \vee \neg\Box^D(\beta \leftrightarrow \Omega^D\beta)))$.

Axiom scheme AX-5: $\Omega^D\alpha \supset \Diamond^D\alpha$.

Axiom scheme AX-6: $(\Box^D\beta \,\&\, \Box^D\Omega^D\beta) \supset \beta$.
Axiom scheme AX-7: $A^D\alpha \supset (\Omega^D[t_k] \leftrightarrow [\Omega^R t_k])$.
Axiom scheme AX-8: $(t_i =+= t_k) \leftrightarrow (G^D[t_i] \leftrightarrow G^D[t_k])$.
Axiom scheme AX-9: $(t_i =+= g) \supset \Box^D G^D[t_i]$.
Axiom scheme AX-10: $(t_i =+= b) \supset \Box^D W^D[t_i]$.
Axiom scheme AX-11: $(G^D\alpha \leftrightarrow \neg W^D\alpha)$.
Definition scheme DF-1: when α is a formula of Φ^{DR}, then $\Diamond^D\alpha$ is a *name* of/for $\neg\Box^D\neg\alpha$.

In AX-1, AX-2, AX-3, AX-4, AX-5, AX-6, and AX-7, the sign Ω^D (belonging to the meta-language of Φ^{DR}) denotes a (any) "*perfection* modality *de dicto*" exclusively. Not all the above-mentioned *de dicto* modalities are called "*perfection* ones". (It is possible to call them merely "perfections *de dicto*".) The set Δ^D of symbols denoting *perfection*-modalities *de dicto* is the following {Ж^D, ©^{D}, K^D, D^D, F^D, C^D, Y^D, P^D, J^D, T^D, B^D, G^D, U^D, O^D, Щ^D, H^D, I^D, $\Box^D$}. Obviously, Δ^D is only a subset of the above-mentioned set of all symbols denoting *de dicto* modalities under consideration in this article. For instance, W^D, S^D, Z^D, and $\Diamond^D$ are signs of such *de dicto* modalities, which are not *de dicto* perfections.

In AX-7, the sign Ω^R (belonging to the meta-language of Φ^{DR}) denotes a (any) "*perfection* modality *de re*" exclusively. The set Δ^R of symbols denoting *perfection*-modalities *de re* is the following {Ж^R, ©^{R}, K^R, D^R, F^R, C^R, Y^R, P^R, J^R, T^R, B^R, G^R, U^R, O^R, Щ^R, H^R, I^R, $\Box^R$}. It is easy to see that there is a *one-to-one corespondence* between elements of the sets Δ^D and Δ^R of perfection modalities.

Evidently, all the above-said is *semantically meaningless*; in any relation to such things, which are external to Φ^{DR}, all the above-given exact definitions make no sense as they are *purely syntactic* ones. However, this is not an outcome of absentmindness (or a delinquency committed by negligence) but such a scientific abstraction which is deliberately accepted on purpose. The intentionaly admitted scientific abstraction is perfectly resonable, when and only when, its adequateness domain is well-defined. Therefore, to make the present article text as a whole perfectly meaningful, now I have to leave the above-defined *syntaxis* of the artificial language of Φ^{DR} for a hitherto completely unknown (absolutely undefined) *semantics* of that artificial language.

1.2. Moving from Syntaxis of the New Formal Multimodal Axiomatic Theory ΦDR to its Semantics

The above-placed section 3.1 of this paper, presents the *purely syntactic* definition of Φ^{DR}, which has been intentionally deprived of its relevant philosophical contents (due to the accepted scientific abstraction). The formal-philosophy axiomatic system Φ^{DR} is a *multimodal* one, but hitherto concrete contents of the modalities under consideration have been revealed not sufficiently; the theory Φ^{DR} has been considered as an exactly *formal* theory. Now, in the given part of the paper, namely, in the section 3.2, I am to relax the formality of Φ^{DR} by shifting immediately to *concrete philosophical contents* of the above-mentioned modalities studied in Φ^{DR}. In the present paper it is implied that semantic meanings of the habitual artificial-language signs of classical symbolic logic are already introduced and well-defined owing to relevant handbooks. As the quite clear semantic meanings of the relevant proper-logic symbols are well-known, it is redundant to define them here. But, such unusual (inhabitual, perhaps, very odd) signs of the artificial language of Φ^{DR}, which are exploited systematically in the proper-philosophy-axioms (ones of epistemology, ontology, etc.), require special introduction and precise definition of their semantic meanings.

Meanings of the lowercase Latin letters q, p, d (and of the same letters possessing lower number indexes) named "*dictum-variables*" are analogous to the meanings of the habitual "*propositional variables*". But there is a substantial difference: in Φ^{DR}, values of "*dictum variables*" belong to the set of *dictums*, to which (set) not only all true or false *sentences* (statements) but also all true or false theories (logically organized systems of propositions) belong. Thus, generally speaking, the dictum-variables range over the set of either true or false dictums. If an interpretation of Φ^{DR} is provided (well-defined), then a *dictum-constant* means (in the given interpretation) quite a definite (perfectly fixed) element from the set of dictums, namely, either a concrete true or false sentence (statement) or a concrete true or false theory.

According to the habital (statistical) linguistic norm (custom rule), from the Latin language, "*dictum*" is to be translated as "an expression of (a thought …) in words", for example, as "a proposition (sentence) *q*", or "an affirmation of (…)". But, there is a heuristically important possibility deliberately to shift from "affirmation of (a proposition …)"

to a *significantly more general* "affirmation of (a proposition …, or a theory …)" as along with uttering separate statements, one can affirm also a theory (logically organized system of statements). As the indicated innovative generalization is accepted, in this paper, it is presumed (as a hypothesis wothy of investigation) that a theory is also a dictum. Hence, attaching *de-dicto*-modalities Ж (existence), C (consistency), T (truth), Y (completeness), and D (decidability) to theories is vindicated in Φ^{DR}. Relevant information of modalities *de-dicto* and *de-re,* along with interesting philosophical discussing their interconnections, can be found, for example, in [Kneale, 1962; Целищев, 1978].

Defining semantic meanings of formal-language-expressions is defining an *interpretation-function*. For defining the interpretation-function, it is necessary to define precisely: (1) such a set which is called "realm (or domain) of interpretation" (hereafter the letter M denotes the set which is the domain of interpretation); (2) an "assessor (valuator)" *V*. If a standard interpretation of Φ^{DR} is fixed, then, by definition, M is such a set, each element of which possesses: (1) one and only one proper *axiological value* belonging to the set {good, bad}, and (2) one and only one proper *ontological value* belonging to the set {exists, not-exists}.

The *axiological variables* (x, y, z, x_k, y_m, z_i, …) take their values from the domain of interpretation (M).

The *axiological constants* "b" and "g" denote abstract *axiological values* "bad" and "good", respectively.

Valuating an element belonging to M by a definite (fixed) assessor V is nothing but ascribing an *axiological value* [either g (good), or b (bad)] to the element. The assessor V may be either an individual or a collective (it does not matter). Certainly, any change of V can result in a change of some (relative) evaluations, nevertheless, such mutations cannot change the set of absolutely immutable formal-axiological laws of two-valued algebra of metaphysics (as formal axiology), which absolutely universal laws are not relative but absolute moral evaluations. The laws in question are *constant valuation-functions* possessing the axiological value "good" under any combination of the values of their arguments. Although V is such a *variable*, values of which belong to the set of all possible assessors (interpreters), any well-defined interpretation of Φ^{DR} necessarily implies that the value of assessor-*variable* V is fixed. Any change of value of V means a change of interpretation.

In the given paper, "e" and "n" denote "… exists" and "… not-exists", respectively. The lowercase Latin letters "e" and "n" are called "*ontological constants*". In any standard interpretation of Φ^{DR}, by definition, one and only one element of the four-element-ed set {{g, n}, {g, e}, {b, n}, {b, e}} corresponds to every element of the domain of interpretation (the above-introduced set M). That is why Φ^{DR} may be considered as a formal semantic representation (discrete mathematical model) of an important truth existing in "Meinong's jungles" [Meinong, 1960], [Russell, 1905; 1941; 1992], [Jacquette, 1996; 1997; 2015], [Marek, 2022], [Parsons, 1980; 1982], [Perszyk, 1993], [Berto, 2012], [Berto, and Priest, 2014], [Routley, 1980], [Zalta, 1983]. The lowercase Latin letters "e" and "n" belong to the alphabet of meta-language. They do not belong to the alphabet of object-language of Φ^{DR}, according to the above-given definition (of the alphabet). Notwithstanding, "e" and "n" are represented in the object-language of Φ^{DR} *indirectly* by means of *square-bracketing*: the *ontological* statement-form "t_i exists" is represented by formula $[t_i]$; the *ontological* statement-form "t_i does not exist" is represented by formula $\neg[t_i]$. This implies that square-bracketing is a very important aspect of precise defining the proper philosophical *semantics* of Φ^{DR}.

From the viewpoint of *formal modal logic* of values, preferences, and assessments, the axiom schemes AX-11 and AX-12 are quite clear and obvious. In contrast to them, the almost unknown (extraordinary, aunhabitual) nontrivial axiom-schemes AX-8, AX-9, AX-10 represent not the symbolic *formal logic of* (evaluative modal) *judgements* but a symbolic *formal axiology* – general theory of abstract-*value-forms of any* (either existing or not-existing) *things*. (This is an option of systematical rationalizing Meinongianism, or a special kind of its being quite consistent.) The concept "symbolic *formal-logic*" is not identical (logically) to the concept "symbolic *formal-axiology*", hence, "formal-logic inconsistency" and "formal-axiological inconsistency" are not synonyms.

In my opinion, the above-said (of "t_i exists" and "t_i does not exist") is an adequate formal semantic representation of the extraordinary doctrine uniting existent and nonexistent objects in one system of philosophical ontology [Fine, 1984; 1985], [Hintikka, 1984], [Jacquette, 1996; 1997; 2015], [Marek, 2022], [Parsons, 1980; 1982], [Perszyk, 1993], [Priest, 2005], [Reicher, 2022], [Smith, 1985], [Zalta, 1983].

N-placed terms of Φ^{DR} are interpreted as *N*-placed *evaluation-functions* defined on the set M. To exclude possible misunderstandings produced by concept confusions, it is worth noting here that the word combination "*evaluation function*" is used in this paper in three *qualitatively different* meanings which are precisely defined as follows.

(I) any function is called an "*evaluation-function*", if and only if the function takes its values from the set of *axiological values* {g (good), b (bad)}.

(II) a function is called "*mixed evaluation-function*", if and only if the function takes values from the set of *axiological values* {g (good), b (bad)}, and its variables take their values from the set M. Hereafter the symbol √ shall stand for the *mixed* evaluation-functions (mappings in the proper mathematical meaning of the word "mapping"): $M^N \rightarrow$ {g (good), b (bad)}.

(III) a function is called "*pure evaluation-function*", if and only if the function takes its values from the set of *axiological values* {g (good), b (bad)}, and variables of the function take their values from the same set.

Speaking of exactly the *pure* evaluation-functions I mean the following mappings:

$\{g, b\} \rightarrow \{g, b\}$, if one speaks of the evaluation-functions determined by *one* evaluation-argument;

$\{g, b\}\times\{g, b\} \rightarrow \{g, b\}$, where "×" stands for the Cartesian product of sets, if one speaks of the evaluation-functions determined by *two* evaluation- arguments;

$\{g, b\}^N \rightarrow \{g, b\}$, if one speaks of the evaluation-functions determined by *N* evaluation-arguments, where *N* is a finite positive integer.

It is well-known that in the two-valued algebraic system, there are exactly 4 *one-placed pure evaluation-functions* and exactly 16 *two-placed pure evaluation-functions*. But it *seems* that this well-known information contradicts logically to the below-located tables 1 and 2 possessing more than 4 columns. The illusion of paradox is a result of the fact that the number of columns is equal to the number of *mixed* evaluation-functions under definition by the tables. Identifying and substituting for each other the *pure* and the *mixed* (evaluation functions) is the cause of/for the paradox illusion to be excluded. The *logically different* notions of the *pure* and the *mixed* (evaluation functions) *must be distinguished* systematically.

The concept "one-placed evaluation-function" is instantiated below by Table 1. (Here it is relevant to note that hereafter the upper literal index R standing immediately after a capital letter implies that this letter stands for such an evaluation-function which is a modality *de re*.) Precise dtfining the interpretation of *de re* modalities as *one-placed evaluation-functions* is accomplished by the below-located tables 1 and 2.

Table 1

Interpretation of *de re* modalities as *unary evaluation-functions*

$\surd x$	$\surd \text{Ж}^R x$	$\surd N^R x$	$\surd \Box^R x$	$\surd \Diamond^R x$	$\surd V^R x$	$\surd ©^R x$	$\surd K^R x$	$\surd P^R x$	$\surd F^R x$	$\surd D^R x$	$\surd G^R x$	$\surd B^R x$
g	g	b	g	g	b	g	g	g	g	g	g	g
b	b	g	b	b	g	b	b	b	b	b	b	b

By Table 1, the one-placed term $\text{Ж}^R x$ is interpreted as *one-placed evaluation-function* "*existence, being of* (what, whom) *x*". The one-placed term $N^R x$ is interpreted as one-placed evaluation-function "*nonexistence, nonbeing of* (what, whom) *x*". The sign $\Box^R x$ stands in standard interpretation for "*necessity of* (what, whom) *x*". The sign $\Diamond^R x$ stands in standard interpretation for "*possibility of* (what, whom) *x*". The symbol $V^R x$ – "*volatility, mutability, changeability of* (what, whom) *x*", or "*possibility of a change of* (what, whom) *x* by an agent, under some conditions". $©^R x$ – "*constructivity* or *constructiveness* of *x*, i.e. *possibility of construction* of *x*". $K^R x$ – "*knowledge of* (what, whom) *x*, i.e. *acquaintance with* (what, whom) *x*. $P^R x$ – "*proof (evidence) of* (what) *x*", or "*basis for accepting x (means of/for forcing to accept) x*". $F^R x$ – "*faith, belief, trust in* (what, whom) *x*". $D^R x$ – "*algorithm of/for construction of* (what, whom) *x*". $G^R x$ – "*positive moral value (goodness) of* (what, whom) *x*". $B^R x$ – "*positive aesthetic value (beauty) of* (what, whom) *x*".

Table 2

Interpretation of *de re* modalities as *one-placed valuation-functions*

$\surd x$	$\surd WRx$	$\surd URx$	$\surd JRx$	$\surd TRx$	$\surd CRx$	$\surd YRx$	$\surd IRx$	$\surd SRx$	$\surd HRx$	$\surd ZRx$	$\surd \text{Щ}Rx$	$\surd \text{Я}Rx$
g	b	g	g	g	g	g	g	b	g	b	g	b
b	g	b	b	b	b	b	b	g	b	g	b	g

By Table 2, the one-placed term $W^R x$ is interpreted as *unary evaluation-function* "*negative moral value* (badness, evilness, wickidness) of (what, whom) *x*". $U^R x$ – "positive utilitarian value (utility) of (what, whom) *x*". $J^R x$ – "*positive hedonistic value (pleasantness) of* (what, whom) *x*", in other words, "*happiness with, or joy from* (what, whom) *x*". $T^R x$ – "*truth of x*", or "*truthfulness (adequateness, rightness, autenticity) of x*". $C^R x$ – "*consistency of x*". $Y^R x$ – "*completeness of x*". $I^R x$ – "*immutability, unchangeability, constantness of* (what, whom) *x*", or "*impossibility of a (any) change of* (what, whom) *x* by an agent". $S^R x$ – "an agent's *sensation of* (what, whom) *x*", or "*feeling* (what, whom) *x* by an agent (under some conditions, either immediately, or by means of some instruments)". ". $H^R x$ – "*imperseptibility of* (what, whom) *x*", or "*x's being imperseptible (sensually unverifyable, inaccessible to the senses)* for/by any agents under any conditions". (In other words, $H^R x$ is interpreted as "impossibility of $S^R x$".) $Z^R x$ – "a *change of* (what, whom) *x* by an agent (either immediately, or by means of some instruments), in some fixed time-and-space, under some special conditions". (Hence, $I^R x$ is interpreted as "impossibility of $Z^R x$.) $\text{Щ}^R x$ – "*essence* of (what, whom) *x*". $\text{Я}^R x$ – "*appearance, phenomenon* of (what, whom) *x*".

Although today logicians are quite reconciled with (and even used to) talks of truth as modality *de dicto* [Wright 1996; Wolenski 2016], they are not used to talks of truth as modality *de re*. (Therefore, $T^R x$ is an odd thing for them.) However, sometimes not only "grass-roots" but also selebrated thinkers use the word "truth" in such meaning which is *de-re*-modality one. Look, for example, into the following wonderful citation from G. W. Leibniz' "On the Ultimate Origination of Things", interesting comments to which (citation) can be found in [Geretto 2016, p. 617-632].

"These considerations must be held to be not only pleasing and consoling, but most true. I think that in the universe nothing is truer than happiness, nor is anything happier or sweeter than truth" [Leibniz 1989, p. 154]. This wonderful citation demonstrates that not only the above-defined (by Table 2) *de re* modaliteies T^R *and* J^R *understood as evaluation-functions,* but also *their* compositions T^RJ^R *and* J^RT^R *have been discussed by* Leibniz. Thus, not only talks of truth as a modality *de dicto* but also talks of truth as a modality *de re* are vindicated.

The concept "*binary evaluation-function*" is exemplified below by Table 3. In the given article, the upper index 2 standing immediately after a capital letter implies that the letter denotes *a binary evaluation-function*.

Table 3

Definition of interpretation of the *binary* evaluation-functions

√x	√y	√N2xy	√K2xy	√S2xy	√E2xy	√V2xy	√A2xy	√X2xy	√T2xy	√Z2xy	√P2xy	√C2xy
g	g	b	g	b	g	b	g	b	b	b	g	g
g	b	b	b	g	b	g	g	b	b	b	g	b
b	g	b	b	g	b	g	g	g	g	g	b	g
b	b	g	b	g	g	b	b	b	b	b	g	g

In Table 3, the two-placed term N^2xy is interpreted as binary evaluation-function "*realizing neither x nor y*"; the two-placed term K^2xy is interpreted as binary evaluation-function "*uniting x* and *y*", or "*joint being* of *x* with *y*", or "*being of both x* and *y* together". The two-placed term S^2xy is interpreted as binary evaluation-function "*split, separation, disengagement, discommunication* between *x* and *y.* E^2xy – "*equivalence (identity of values)* of *x* and *y*". V^2xy – "*choosing and realizing* such and *only such* an element of the set {*x, y*}, which is: (1) *the best* one, when both *x* and *y* are good; (2) the least bad one, when both *x* and *y* are bad; (3) the good one, when *x* and y have opposite values. (Thus, V^2xy means an *excluding choice and realization of only the optimal* between *x* and *y*.) The term A^2xy is interpreted as the binary evaluation-function "realizing a *not-excluding-choice* result, i.e. (1) *realizing* K^2xy *when both x and y are* *good*, and (2) *realizing* V^2xy *otherwise*". X^2xy – evaluation-function "joint being of *y* with nonbeing of *x*", or "*y's being without x*". T^2xy – "*termination* of *x* by *y*". Z^2xy – "*y*'s *contradiction* to (with) *x*". P^2xy – "*preservation, conservation, protection* of *x* by *y*". C^2xy – evaluation-function "*y's existence, presence in x*". Additional instantiations of the concept "*binary evaluation-function*" can be found in [Лобовиков 2007; Lobovikov, 2022].

For excluding possible psychological-linguistic illusions and conceptual confusions, now it is quite opportune and perfectly relevant to highlite that in a standard interpretation of Φ^{DR}, the symbols $Ж^Rx$, N^Rx, B^Rx, U^Rx, J^Rx, N^2xy, K^2xy, S^2xy, A^2xy denote not some predicates but some *n*-placed *evaluation-functions*. When a standard interpretation of Φ^{DR} is provided, then such object-language expressions of Φ^{DR}, which (expressions) have the forms (t_i=+=t_k), (t_i=+=g), (t_i=+=b), represent some *predicates* in Φ^{DR}.

According to semantics of Φ^{DR}, when t_i is a term of Φ^{DR}, then, if a formula of Φ^{DR}, possessing the form [t_i], is interpreted, then the mentioned formula represents (in the given interpretation) an *either false or true proposition* having the form "t_i exists". Thus, according to the definition, in any standard interpretation, any formula [t_i] is true then and only then, when t_i possesses the *ontological value* "e (exists)" in the given interpretation. Also, in any standard interpretation of Φ^{DR}, any formula [t_i] is false, then and only then, when t_i possesses the ontological value "n (not-exists)" in the given interpretation.

By definition of semantics of Φ^{DR}, in a standard interpretation of Φ^{DR}, the formula scheme (t_i =+= t_k) is a proposition possessing the form "t_i is *formally-axiologically equivalent* to t_k"; this proposition is true if and only if (in that interpretation) the terms t_i and t_k obtain identical *axiological values (*from the set {good, bad}) under any possible combination of *axiological values* of their *axiological variables*. In other words, by definition, (t_i =+= t_k) iff ($\surd t_i = \surd t_k$).

By definition of semantics of Φ^{DR}, in a standard interpretation of Φ^{DR}, the formula scheme (t_i =+= b) is a proposition having the form "t_i is a *formal-axiological contradiction*" (or "t_i is *formally-axiologically, or invariantly, or absolutely bad*"); this proposition is true if and only if (in that interpretation) the term t_i acquires axiological value "bad" under any possible combination of axiological values of the axiological variables. In other words, by definition, (t_i =+= b) iff ($\surd t_i$ = b).

By definition of semantics of Φ^{DR}, in a standard interpretation of Φ^{DR}, the formula scheme (t_i =+= g) is a proposition having the form "t_i is a *formal-axiological law*" (or "t_i is *formally-axiologically, or invariantly, or absolutely good"*); this proposition is true if and only if (in the interpretation) the term t_i acquires *axiological value* "good" under any possible combination of axiological values of the axiological variables. In other words, by definition, (t_i =+= g) iff ($\surd t_i$ = g).

Concerning the above-given definition of sematic meaning of (t_i =+= t_k) in Φ^{DR}, it is indispensable to highlight the important linguistic fact of homonymy of the words "is", "means", "implies", "entails", "equivalence" in natural language. On the one hand, in natural language, these words may have the well-known formal logic meanings. On the other hand, in natural language, the same words may stand for the above-defined *formal-axiological-equivalence* relation "=+=". This ambiguity of natural language is to be taken into an account; the different meanings of the homonyms are to be separated systematically; otherwise the homonymy can head to logic-linguistic illusions of paradoxes.

Owing to the above-provided definition of proper philosophical semantics (formal-ontological-and-formal-axiological one) of/for the formal theory Φ^{DR}, readers can easyly notice that the above-defined algebraic system of metaphysics as formal axiology plays the role of such a *theory of relativity* of assessments, in which (relativity theory), the *laws* (*formal-axiological* ones) of that algebraic system are nothing but constantly-good evaluation-functions. Speaking differently, the absolutely universal and immutable laws of valuation-relativity are *invariants* in relation to all possible changes of valuator (interpreter) V.

Systematically using the above-provided definitions, one can generate (and examine) the following philosophically interesting formal inferrence in Φ^{DR} from the assumption $A^D\alpha$.

1) $A^D\alpha \supset (\Omega^D[t_k] \leftrightarrow [\Omega^R t_k])$: axiom scheme AX-7.
2) $A^D\alpha$: assumption.
3) $(\Omega^D[t_k] \leftrightarrow [\Omega^R t_k])$: from 1 and 2 by modus ponens.

4) $\Box^{D}[x] \leftrightarrow [\Box^{R}x]$: from 3 by substituting: $\Box$ for Ω; *x* for t_k. Into the natural language, 4) is translated as follows: it is necessary that *x* exists, if and only if there is a necessity of *x*.

5) $K^{D}[x] \leftrightarrow [K^{R}x]$: from 3 by substituting: K for Ω; *x* for t_k. Into the natural language, 5) is translated as follows: an agent knows that *x* exists, iff the agent's knowledge of *x* exists.

6) $F^{D}[x] \leftrightarrow [F^{R}x]$: from 3 by substituting: F for Ω; *x* for t_k. Into the natural language, 6) is translated as follows: an agent believes that *x* exists, iff there is the agent's belief in *x*.

7) $D^{D}[x] \leftrightarrow [D^{R}x]$: from 3 by substituting: F for Ω; *x* for t_k. Into the natural language, 7) is translated as follows: there is an algorithm of/for deciding that (*x* exists), iff there is an algorithm of/for implementation (construction) of *x*.

8) $G^{D}[x] \leftrightarrow [G^{R}x]$: from 3 by substituting: G for Ω; *x* for t_k. Into the natural language, 8) is translated as follows: it is (morally) good that (*x* exists), iff a moral goodness (positive moral value) of *x* exists.

9) $B^{D}[x] \leftrightarrow [B^{R}x]$: from 3 by substituting: B for Ω; *x* for t_k. Into the natural language, 9) is translated as follows: it is beautiful that (*x* exists), iff a beauty (positive aesthetic value) of *x* exists.

10) $U^{D}[x] \leftrightarrow [U^{R}x]$: from 3 by substituting: U for Ω; *x* for t_k. Into the natural language, 10) is translated as follows: it is useful that (*x* exists), iff a use (utility) of/from *x* exists.

11) $J^{D}[x] \leftrightarrow [J^{R}x]$: from 3 by substituting: J for Ω; *x* for t_k. Into the natural language, 11) is translated as follows: it is a pleasure, joy, happiness that (*x* exists), iff there is a happiness, joy, pleasure of/from *x*.

In my opinion, from the viewpoint of formal philosophy proper, the above-considered (hitherto never published elsewhere) option (direction) of investigating interconnection of *de dicto* and *de re* types of modalities is worth developing further.

1.3. Conditions of Truth of/for Formulae of ΦDR in Standard Model of/for It

Concerning the important problem of proper-logic *sematic* status (*truth*-related one) of moral-legal *evaluations*, i.e. statements of moral-legal *value* (which *evaluations* are also *either good or bad moral-legal* *acts* according to the bivalent *ethics and theory of the natural law*), it is worth taking into an account formal-*axiological* equivalences ($Ж^R x$ =+= x) and ($C^R x$ =+= x), in which meanings of *terms x,* $Ж^R x$ and $C^R x$ belong to the above-defined set M (named "interpretation domain"). When an interpretation of $Φ^{DR}$ is defined, then every element of M obtains one and only one of the values belonging to the four-elemented set of two-element-ed sets {{g, n},{g, e}, {b, e}, {b, n}}. Probably, it is worth recalling here that: "g" stands for "good"; "b" stands for "bad"; "e" stands for "exists"; "n" stands for "not exists".

From the above-defined viewpoint, it is evident that formal-*axiological* equivalences ($Ж^R x$ =+= x) and ($C^R x$ =+= x) are essentially *similar* (significantly *analogous*) to the well-known formal-*logical tantamount-ness* ((It is true that p) $\equiv p$), considered systematically in the logical semantics by Tarsky. However, in this relation, it is very important to take into an account that, strictly speaking, *analogy* (*similarity*) *is not an identity* relation. Generally speaking, *similarity (analogousness) is not transitive*, hence, it is not a relation of equivalence. According to the logical positivism, strictly speaking, *statements of values* (aesthetic, religious, moral-legal) *are neither true nor false*, hence, in relation to *proper logic* semantics (i.e. *logic as a theory of truth*), they are semantically *meaningless*.

However, $Φ^{DR}$ is not a *pure* formal *logic* theory; it is not *only* logic system. $Φ^{DR}$ is a result of *application* of formal logic to what is not formal logic but possesses *own proper* axioms, for instance, epistemic, axiological ones.

Moreover, $Φ^{DR}$ is a *multimodal* theory (truth and falsity are considered in it along with many other substantially different kinds of modalities, for instance, along with the *evaluative* modalities (moral *goodness*, *wickedness*, etc.).

To avoid possible misinterpretations of the below-placed formulations, here it is worth emphasizing that "t" possessing no indexes stands in the present paper for "true", and "t" possessing a lower index (literal one) stands for a *term*.

In logic semantics of the above-defined artificial language of $Φ^{DR}$, truth of moral-legal *judgement* $G^D[t_i]$ is quite exactly defined by the following *formal-logical equivalence*: ((assessment $G^D[t_i]$ has proper *logic* value "t") ≡ (term t_i has *axiological* value "g")). The artificial symbol "≡" stands here for the habitual relation of *formal-logical tantamount-ness* (coincidence of proper logic values). This *tantamount-ness* is a quite precise fundamental *definition* of proper logic semantic meaning of $G^D[t_i]$ in the language of Φ^{DR}.

Keeping in mind all the above-given definitions, conditions of truth of/for formulae of Φ^{DR} (in its standard model) are defined in the present paper as follows:

I. ((formula $[t_i]$ possesses proper *logic* value "t") ≡ (term t_i possesses *ontological* value "e")).

II. ((formula $G^D[t_i]$ possesses proper *logic* value "t") ≡ (term t_i possesses *axiological* value "g")).

III. ((formula $W^D[t_i]$ possesses proper *logic* value "t") ≡ ⌉(term t_i possesses *axiological* value "g")).

IV. For any formula α of Φ^{DR}, formula $K^D\alpha$ possesses proper *logic* value "t", if and only if, either it is true that $E^D\alpha$, or it is true that $A^D\alpha$. Here, "either …, or …" stands for the strict (excluding) disjunction. Consequently, it is quite relevant to use the below-placed hexagon containing the logic opposition square for visualizing (modelling geometrically) the system of proper formal-logic semantic (truth-related) logical interrelations among *de dicto epistemic* modalities K^D, A^D, E^D, while precisely defining conditions of/for truth of formulae of Φ^{DR} in standard model this theory.

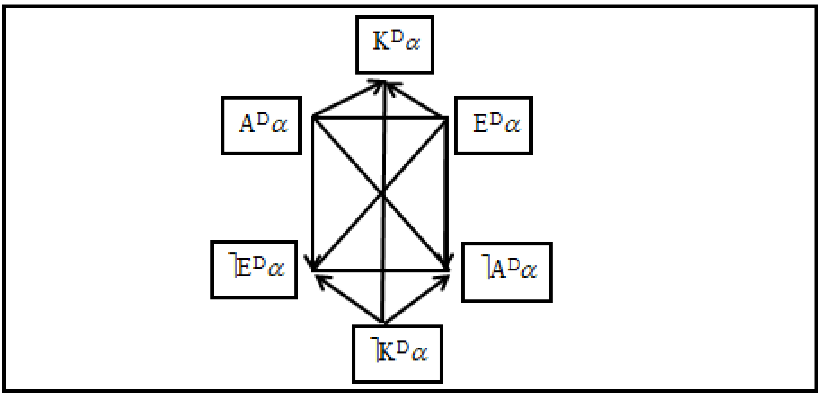

Fig. 1. The logical opposition-square-and-hexagon of epistemic modalities *de dicto*

According to the so-called "*traditional* formal logic", in the above visual image, the upper horizontal line represents geometrically the well-known *contrariety* relation; in its turn, the bottom horizontal line in Fig. 1 represents geometrically the well-known *sub-contrariety* relation; the three lines crossing the square geometrically model the well-known *contradictoriness* relations. The arrows model visually the well-known relations of *subordination* (formal-logic *consequence*). With respect to the epistemic modalities *de dicto*, all the truth-related formal-logic rules of the well-known traditional formal-logic opposition square are valid. The heuristically and pedagogically fruitful idea of geometric modelling proper formal-logic aspect of conceptual systems by the square of conceptual opposition has been known and exploited systematically since ancient times to our ones. Nowadays, the traditional square of opposition is essentially generalized by the hexagon and many *qualitatively novel* interpretations of/for the old idea (of visual geometric modeling proper logic relations and rules) have been invented and exploited.

According to the given paper, in standard model of Φ^{DR}, conditions of truth of/for formulae containing the formula $E\alpha$, are exactly defined by the following statements:

V. For every formula α, when it is true that $E^{D}\alpha$, then it is also true that $K^{D}\alpha$.

VI. For every formula α, when it is true that $\Diamond^{D}S\alpha$, then it is also true that $E^{D}\alpha$. This represents (in truth-related semantics of/for Φ^{DR}) the positivist requirement of science methodology emphasizing that proper *empirical* knowledge is to be *sensually verifiable* either actually or potentially (in principle). Obviously, the suggested representation (model) of the sensual verification-ism substantially differs from its original actually realized in history of philosophy of science (here I mean the substantial difference of *sufficient* and *necessary* conditions of/for quite rational affirming that knowledge is actually *empirical one*.

VII. For every formula α, when it is true that $\Diamond^{D}\neg\alpha$, then it is also true that $E^{D}\alpha$. This represents (in truth-related semantics of/for Φ^{DR}) exactly that positivist principle of science methodology (i.e. indispensable condition of/for scientific-ness of knowledge), according to which (principle), *actually scientific* (=empirical) knowledge has to be *falsifiable* either actually or potentially (in principle). Evidently, the suggested representation (model) of the falsification-ism substantially differs from its original actually realized in history of philosophy of science. Here I imply the noteworthy difference between *sufficient* and *necessary* conditions of/for *empirical-ness* of knowledge (the indicated situation is *analogous*, i.e. *different but similar*, to the one described in the above-formulated item VI). Special attention to the potential falsifiability of proper scientific knowledge had been paid in XX century. However, some outstanding specialists in science philosophy of XX century have systematically criticized the overly obsessive (pseudo-rationalistic) tendency to oust and replace the limitedly respectable criterion of *sensual verifiability* (potential verification by sensation) with the limitedly respectable criterion of *falsifiability* (potential refutation), in rational philosophy, methodology and logic of science proper. The opponents reasonably pointed out that, according to history and philosophy of science, neither verifiability, nor falsifiability, nor their non-excluding disjunction are *necessary* conditions of/for actually *empirical* (=proper *scientific*) knowledge; the stormy discussion implies a nontrivial hypothesis that, perhaps, there are also some additional not yet recognized nontrivial principles (criterions) of perfect empirical-ness (actual scientific-ness) of human knowledge. Which still not well-recognized principles (criterions) are meant here? In this paper, a nontrivial hypothetic answer to this hard question is given by the below-placed statement VIII representing also a *sufficient* (but also a not necessary) condition of/for quite rational affirming actual *empirical-ness* (=scientific-ness) of human knowledge.

VIII. For any formulae α and ω of Φ^{DR}, if it is possible that proper logic values of the expressions (formula $\Omega^{D}\omega$ possesses logic value "t") and (formula ω possesses logic value "t") are different, then it is true that $E^{D}\alpha$. Here it is worth recalling that Ω^{D} stands for an (arbitrarily taken) element of the set of *de dicto* perfection-modalities which (set) is defined above. In standard model of Φ^{DR}, the *necessary and sufficient* condition (truth-related one) of *empirical-ness* of knowledge is the below-formulated condition IX.

IX. For every formulae α and ω, when and only when, in a standard model of Φ^{DR}, it is true that $E^{D}\alpha$, then, in the standard model of Φ^{DR}, either (1) it is true that $\Diamond^{D}\rceil\alpha$, or (2) it is true that $\Diamond^{D}S\alpha$, or (3) it is true that it is possible that it is false that proper logic values (truth-ones) of formulae ω and $\Omega^D\omega$ are identical.

Taking the above-formulated conditions VIII and IX seriously, one can discover (notice and recognize) that, generally speaking, knowledge can be actually *empirical* one even then, when it is neither sensually *verifiable* nor actually *falsifiable*. Being psychologically unexpected this discovery destroys the habitual paradigm as it goes far beyond traditional tenets of the classical empiricism. Nevertheless, this substantial reconstruction of precise formulation of the criterion (=*necessary-and-sufficient*-condition) of *empirical-ness* of knowledge makes it possible easily to understand and simply to explain some hitherto mysterious facts of cognition history.

Truth-conditions for formulae of Φ^{DR}, containing the expression $A^D\alpha$, are defined in this article as follows:

X. For every formula α of Φ^{DR}, when it is true that $A^D\alpha$, then it is true that $K^D\alpha$.

XI. For every formula α of Φ^{DR}, when it is true that $A^D\alpha$, then it is true that α.

XII. For every formula α of Φ^{DR}, when it is true that $A^D\alpha$, then it is true that $\Box^D\alpha$.

XIII. For every formula α of Φ^{DR}, when it is true that $A^D\alpha$, then it is true that $\neg\Diamond^D S\alpha$.

XIV. For every formula α of Φ^{DR}, when $A^D\alpha$, then for every formula ω of Φ^{DR}, the formal-logic tantamount-ness $((\Omega^D\omega) \equiv \omega)$ is valid, or, in other words, ((formula $\Omega^D\omega$ possesses logic value "t"), when and only when (ω possesses logic value "t")), where symbol Ω^D stands for any (arbitrarily taken) element of the above-defined set of *de dicto* perfection-modalities.

XV. For every formula α of Φ^{DR}, when $A^D\alpha$, then, for every formula ω of Φ^{DR}, it is true that $(\Xi^D\omega \equiv \Omega^D\omega)$, where symbols Ξ^D and Ω^D stand for any (arbitrarily taken) elements of the above-defined set of *de dicto* perfection-modalities.

XVI. For every formula α of Φ^{DR}, when $A^D\alpha$, then, for every modality Ω and every term t_k of Φ^{DR}, it is true that proper logic values (truth ones) of $\Omega^D[t_k]$ and $[\Omega^R t_k]$ are identical (do coincide).

The sentence {If $A^D\alpha$, then, for every term t_i, ((formula $T^D[t_i]$ possesses logic value "t") ≡ (formula $[t_i]$ possesses logic value "t"))} is nothing but a concrete *particular* case (instance) of the above-formulated condition XIV.

The below-located statements a) and b) are quite representative particular cases (instances) of the condition XV:

a) If $A^D\alpha$, then, for every formula ω of Φ^{DR}, it is true that $(T^D\omega \equiv C^D\omega)$, where "$T^D$" stands for *de dicto* modality "it is *true* that …", while "C^D" stands for *de dicto* modality "it is *consistent* that …".

b) If $A^D\alpha$, then, for every formula ω of Φ^{DR}, it is true that $(Ж^D\omega \equiv C^D\omega)$, where "$Ж^D$" stands for the *existence de dicto* modality, while "C^D" stands for the *consistency de dicto* modality.

Keeping all the above-said in mind, one can arrive to the conclusion that, when the truth condition of/for $A^D\alpha$ is fulfilled (certainly, this case is *extraordinary* one), then the notorious *modal collapse* happens, namely: *all* the perfection-modalities *de-dicto* are formally-logically tantamount to each other and, consequently, may be resolutely eliminated from relevant expressions; without any change of truth-values of these expressions, hence, under this *extraordinary* condition, Φ^{DR} turns from the *multimodal* theory into the classical propositional calculus which is consistent and complete one. It is an almost common belief that deductive provability of the modal collapse is a grave defect (existentially significant flaw) of/for every formal axiomatic theory of modalities. In other words, when the modal collapse is a theorem of a general theory of modalities, then the general theory of modalities ceases to exist as such. Evidently, this is really so, if one speaks in general. But, it is worth emphasizing here that, generally speaking, the formula implying modal-collapse is not deductively provable in Φ^{DR}. The modal collapse is formally-logically (deductively) derivable in Φ^{DR} *not in general*, but only from the very strong *assumption* (extraordinary *hypothesis*) that $A^D\alpha$. If $E^D\alpha$, then the modal collapse does not exist in Φ^{DR}. Consequently, in general, as a whole, Φ^{DR} is free of the modal-collapse paradox (while, in that *extraordinary* particular case, when $A^D\alpha$, the modal collapse is neither a flaw nor a paradox but quite a *normal* condition).

I faith that, in future, the powerful tendency to multimodality in philosophical logic (in particular, the strong propensity to think of many-ness of qualitatively different kinds of *de dicto* modalities) instantiated in this paper by formulae $G^D a$, $T^D a$, $C^D a$, $Ж^D a$, shall lead philosophers to a significantly clearer recognition of the inevitable ambiguity, polysemy, and essential many-valued-ness of natural human language semantics. The given paper is the start of/for promising scientific investigations in the indicated direction.

1.4. Utilizing the Logical Square of Concept Opposition and Blanché Hexagon as Graphic Models (Means of/for Visualization) of formal-logical interconnections among perfection-modalities de-dicto and of formal-axiological interconnections among perfection-modalities de-re, in the Formal Philosophy System ΦDR

One of theoretically interesting options and prospective directions of investigating *perfection-modalities de-dicto* and *de-re*, is attempting to utilize the hexagon [Blanché 1966] containing the square of opposition for graphic modelling (visualizing) the systems of formal interconnections among the modalities belonging to the two qualitatively different types under consideration. In [Lobovikov 2024, p. 34], the opposition square included into Blanché hexagon has been used for graphic modelling (visualizing) the systems of *formal-logical* interconnections among the *epistemic* modalities *K, A, E*, which are *de dicto* ones. Hereunder I suggest quite a new *formal-logical* interpretation of the geometric figures. The *formal-logic*-square-and-hexagon is exploited below as a *graphic model (means of/for visualization) of formal-logical* interconnections among *any* possible *perfection-modalities de-dicto.*

With respect to *all* perfection-modalities *de dicto*, all the truth-related formal-logic rules of the traditional formal-logic-square-and-hexagon are valid. Fig. 2 models *proper logic* aspect of *de dicto* modal notion systems. According to [Inwagen, Sullivan, and Bernstein 2023], it is highly likely that discourses of *de-dicto* modalities belong to the subject-matter of logic, while discourses of *de-re* modalities belong to the subject-matter of metaphysics. If this is really so, then, in the present article, it is worth undertaking a hypothetic-deductive investigation of an extraordinary possibility of inventing (intentional constructing by analogy) a hitherto never considered *formal-*

axiological square-and-hexagon for modeling graphically not formal-logical but formal-axiological (i.e. proper metaphysical) aspect of relations among *de re* modalities (objects of metaphysics proper).

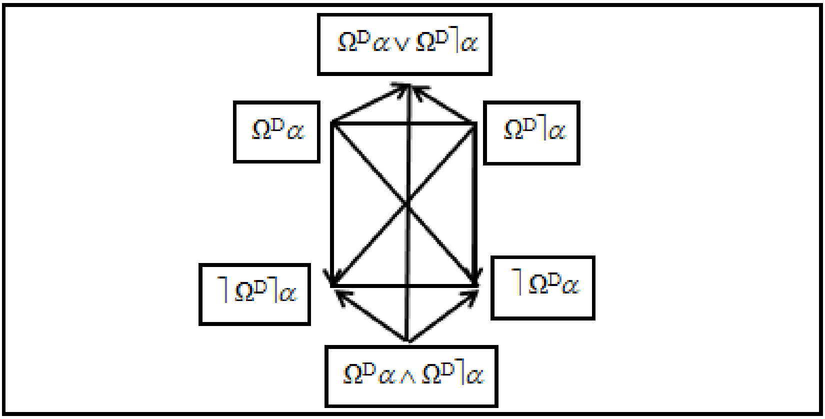

Fig. 2. The *multimodal* square and hexagon of opposition visually modeling *formal-logical* interconnections among *any* perfection-modalities *de-dicto*[1]

If the statement that *de re* modalities belong to the subject-matter of metaphysics is true, then, if the above-mentioned challenging *formal-axiological* conception of metaphysics [Лобовиков 2007; Lobovikov 2022] is accepted (at least as a hypothesis worthy of investigation), then the relations among modalities *de re* are to be *formal-axiological* ones (*formal-axiological* consequence, *formal-axiological* contradiction, etc.).

I think that the pedagogically and heuristically fruitful idea of graphic modelling abstract relations among abstract objects in abstract systems of any nature (well-known and used systematically since ancient times) can be exploited successfully (by analogy) also in connec-

[1] Probably, it is worth recalling here that, in harmony with standard handbooks of the so-called traditional formal logic, in Fig. 1, the bottom horizontal line represents graphically the *sub-contrariety* relation; the upper horizontal line represents visually the *contrariety* relation; the *formal-logic-consequence* relations are graphically modeled by arrows; the lines crossing the square model graphically the *contradictoriness* relations.

tion with the *formal-axiological* systems. This abstract hypothetical statement is exemplified below by Fig. 3, in which square and hexagon model graphically not the habitual proper *formal-logical* relations but unhabitual (qualitatively new) *formal-axiological* ones among *modalities-de-re as evaluation-functions*. To exclude possible misunderstanding, here it is worth highlighting that, in Fig. 3, the artificial language expressions $\Omega^R t_k$, $N^R\Omega^R t_k$, $\Omega^R N^R t_k$, $K^2\Omega^R t_k\Omega^R N^R t_k$, and other mean not "formulae" but "terms" of the theory Φ^{DR}. Thus, in Fig. 3, the lines visually model the relations not among formulae but among terms of Φ^{DR}.

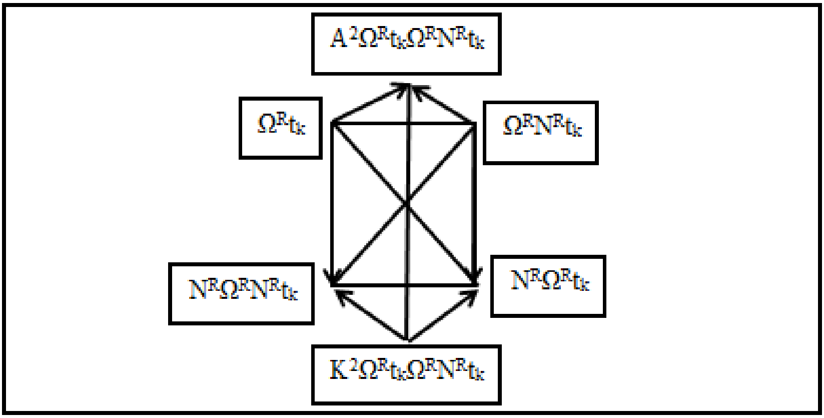

Fig. 3. The *multimodal* square and hexagon of opposition visually modeling *formal-axiological* interconnections among *any* perfection-modalities *de-re*

To exclude possible misunderstanding, it is necessary to emphasize here that, in Fig. 3, arrows model graphically the relations of "*formal-axiological* consequence"; the lines crossing the square model graphically the relations of "*formal-axiological* contradictoriness"; the upper horizontal line represents visually the relation *of* "*formal-axiological* contrariety"; the bottom horizontal line represents graphically the relation *of* "*formal-axiological* subcontrariety". From the psychological point of view, it is highly likely that people can mistake the "*formal-axiological*" relations for the corresponding "*formal-logical*" ones. However, from the proper logic and theoretical-philosophy points of view, this gravely dangerous mistake must be excluded due to perfect recognizing that "*formal-axiological*" and "*formal-logical*" are not synonyms: they have not identical but qualitatively different meanings. (In this connection, readers are advised accurately to compare the relevant definitions, which are strikingly *analogous* but *not identical*; and recognizing *their difference is very important* for dismissing possible illusions of paradoxes. I recall that relevant precise definitions of "*formal-axiological* contradiction", "*formal-axiological* equivalence", et al are provided above in this article.)

The *multimodal* opposition squares and hexagons represented above by Fig. 2 and Fig. 3 have been never published before. Certainly, various qualitatively different concrete interpretations (very interesting and *important* particular cases) of Fig. 2 have been already either discovered (contingently found) or deliberately invented (intentionally constructed) much earlier, especially by Robert Blanché and Jean-Yves Béziau [Blanché1966; Béziau 2003; 2012a; 2012b; Béziau, Buchsbaum, Rey 2018]. But the *multimodal* graphic scheme located in Fig. 2 visually represents for the first time *a universal* of/for the mentioned interesting and important *particular* cases: the *alethic* modalities make up an important *particular* case of Fig. 2; the *deontic* modalities make up an important *particular* case of Fig. 2 as well, and so on; the *volatile* vistas are *potentially infinite* in the indicated respect. This is promising original fruitful applications of the *universal* (visually modeled by the figures) to unexpected contexts and problems to be encountered in future.

Discussion and Exemplification of the Novel Results

Let us begin with discussing the equivalence ($\Box^D[x] \leftrightarrow [\Box^R x]$) formally derived above (in Φ^{DR}) from the presumption of *a-priori*-ness of knowledge. While talking and writing of ontology in physics and metaphysics, thinkers deal with two kinds of necessity: *de dicto* and *de re*. The sentence "It is *necessary that* there is conservation of energy" exploites the *necessity de dicto*, while the sentence "There is *necessity of* conservation of energy" exploites the *necessity de re.* Let us fix and discuss such an *interpretation* of ($\Box^D[x] \leftrightarrow [\Box^R x]$), in which *conservation-of-energy* is a value of the variable *x*. In this fixed interpretation, [x] is a dictum (proposition) "*conservation-of-energy exists*", and $\Box^D[x]$ is a *de dicto* modal proposition (modal statement about the dictum [x]). Along with this, in the fixed interpretation under discussion, $\Box^R x$ *is not a dictum* but a physical *object*: either a physical state of matter, or a material process, or a material thing, i.e. *re*. Both x and $\Box^R x$ belong to the material world: they are not dictums. Consequently, in the concrete interpretation under discussion, $[\Box^R x]$ is a *de re* modal proposition – modal statement about what exists (in the real world). It is easy to see that, in the given interpretation, the equivalence ($\Box^D[x] \leftrightarrow [\Box^R x]$), is true. Hence, with respect to physics, the axiom-scheme AX-7 (of Φ^{DR}) looks quite adequate.

According to history of philosophy, exactly physics had been "native land" (basic professional education) of/for plenty of positivist-minded philosopers negatively treating metaphysics, axiology, and theology. In this connection, it is curious that the equivalence ($\Box^D[x] \leftrightarrow [\Box^R x]$), is true not only in proper physical but also in proper thelogical interpretation. If such a thelogical interpretation of ($\Box^D[x] \leftrightarrow [\Box^R x]$), is suggested, in which a name of God is substituted for *x*, then, being theologically interpreted, ($\Box^D[x] \leftrightarrow [\Box^R x]$) is translated into the natural language by the following biconditional: (it is *necessary that* (God exists)), if and only if ((*necessity of* God) exists). Here, (necessity of God) is not a proposition but a *re*. Certainly, from the classical theology viewpoint, the biconditional is true.

Now let us consider the sentence *"In God we trust"* located on the green banknotes. If it was written *"We trust that (God exists)"*, then *"trust (that ...)"* would be a modality *de dicto.* But the *"trust in (what, whom)"* is a modality *de re*. In the sentence *"There is (our trust in God)", or "(Our trust in God) exists"*, God is considered as a Divine *Re*. For true believers, it is certain that the biconditional *"We trust that (God exists), if and only if (Our trust in God) exists"* is true. This very special biconditional sentence is a concrete exemplification of the abstract equivalence ($F^D[t_k] \leftrightarrow [F^R t_k]$). The true positivists hating talks about God and ignoring *de re* modalities (as metaphysical ones) shall be not happy with the equivalence ($F^D[t_k] \leftrightarrow [F^R t_k]$) exemplified theologically. But, in my opinion, for unpedjudiced formalist-minded philosopers and abstractly thinking logicians this quite formal *equivalence of de dicto and de re biliefs* (not necessarily having theological interperetation) is of some professional interest.

In mathematical logic proper, there are quite natual and fruitful discourses of *provability as modality* [Кузнецов и Муравицкий 1980; Boolos and Sambin 1991]. However, as a rule, the modality discourses of provability are reduced to *provability as modality de dicto*, exclusively. In addition to these writings on proper logic, let us try to consider evidence (proof) and proving (making evident) as modalities *de re*. Obviously, this is something unusual (not habitual) for logicians proper, but lawyers (especially the ones taking part in litigations) are used to the so-called "*real* evidences". For litigants, judges, and investigators, "proving thing (*re*)" has value. Hence, *axiology* (dealing with moral-legal value) is quite relevant to *evidence (proof) as modality de re*. Often, during legal trials, making up (or ruining) a proof (finding or destroying an evidence) are operations with objects (things); in this relation, evidences (proofs) are *objects* of an activityof appropriate lawyers, hence, in this relation, proofs (evidences) are nothing but things (*re*), to which the activity is applied.

Let us consider the following interpretation: term *d* is interpreted as John Fitzgerald Kennedy; term *h* is interpreted as Lee Harvey Oswald; the two-placed term (applied to *d* and *h*) is interpreted as (termination of J. F. Kennedy by L. H. Oswald). In this interpretation, term T^2dh means the act of termination of Kennedy by Oswald, which act is (certainly) not a proposition but an event (*re*). (Here it is worth recalling that the *evaluation-function* T^2xy – "*termination* of *x* by *y*" is defined above by Table 2.) In the interpretation under consideration, term P^RT^2dh (made by attaching P^R to term T^2dh) means *de-re-*modality-term "*evidence (proof) of* (what) T^2dh". Thus, in the interpretation under discussion, terms *d, h,* T^2dh, P^RT^2dh are not propositions (dictums), but things (res): persons, actions, events, states of affairs, real evidences, which are not proper logic phenomena. Nonthеless, in the fixed interpretation, $[P^RT^2dh]$ denotes the either true or false statement (dictum) that there is an evidence that Kennedy has been killed by Oswald. In its turn, $[T^2dh]$ is also an either true or false proposition (dictum), namely, the statement that Kennedy has been killed by Oswald. Consequently, both formulae $[T^2dh]$ and $[P^RT^2dh]$ belong to the subject-matter of logic proper. According to the above-given definition of "formula of Φ^{DR}", if $[T^2dh]$ is a formula, and P^D is a symbol denoting a de-dicto-modality, then $P^D[T^2dh]$ is a formula of Φ^{DR} as well. Usual content analysis and intuition have nothing against proclaiming that $(P^D[T^2dh] \leftrightarrow [P^RT^2dh])$ is true. Thus, the above-considered concrete interpretation makes up an exemplification of the abstract equivalence $(P^D[t_k] \leftrightarrow [P^Rt_k])$.

1. *A Proof of Consistency of* Φ^{DR}

First of all, let us leave the above-presented *axiom-schemes* of Φ^{DR} for the below-located *proper-axioms* of Φ^{DR}.

Axiom AX*-1: $A^Dp \supset (\Box^Dq \supset q)$.

Axiom AX*-2: $A^Dp \supset (\Box^D(p \supset q) \supset (\Box^Dp \supset \Box^Dq))$.

Axiom AX*-3:
$A^Dp \leftrightarrow (K^Dp \;\&\; (\neg\Diamond^D\neg p \;\&\; \neg\Diamond^DSp \;\&\; \Box^D(q \leftrightarrow \Box^Dq)))$.

Axiom AX*-4: $E^Dp \leftrightarrow (K^Dp \;\&\; (\Diamond^D\neg p \vee \Diamond^DSp \vee \neg\Box^D(q \leftrightarrow \Box^Dq)))$.

Axiom AX*-5: $\Box^Dp \supset \Diamond^Dp$.

Axiom AX*-6: $(\Box^Dq \;\&\; \Box^D\Box^Dq) \supset q$.

Axiom AX*-7: $A^Dp \supset (\Box^D[x] \leftrightarrow [\Box^Rx])$.

Axiom AX*-8: $(B^Rx{=}{+}{=}J^Rx) \leftrightarrow (G^D[B^Rx] \leftrightarrow G^D[J^Rx])$.

Axiom AX*-9: $(B^Rx{=}{+}{=}g) \supset \Box^DG^D[B^Rx])$.

Axiom AX*-10: $(B^Rx{=}{+}{=}b) \supset \Box^DW^D[B^Rx])$.

Axiom AX*-11: $(G^Dp \leftrightarrow \neg W^Dp)$.

Дефиниция DF*-1: $\Diamond^Dp$ есть *сокращенное название* для $\neg\Box^D\neg p$, т.е. $(\Diamond^D\omega \leftrightarrow \neg\Box^D\neg\omega)$, по определению.

Now let us consider a function # called "an *interpretation* of the axiom system" and defined precisely by the following list of items 1)–26) as a whole. (Here it worth noting that, in this part of the article, the symbol "t" stands for "true" and the symbol "f" stands for "false".)

1) For any formulae δ and λ of the formal theory Φ^{DR}, and for any binary classical logic connective $\otimes$, it is true that $\#(\delta \otimes \lambda) = (\#\delta \otimes \#\lambda)$.

2) For any formula δ of the formal theory Φ^{DR}, it is true that $\#\rceil\delta = \rceil\#\delta$. (Here the symbol "$\rceil$" stands for the classical-logic unary-operation called "negation".)

3) $\#A^{D}p = f$.

4) $\#\Box^{D}q = f$.

5) $\#q = t$.

6) $\#p = t$.

7) $\#\Box^{D}(p \supset q) = f$.

8) $\#\Box^{D}p = f$.

9) $\#K^{D}p = t$.

10) $\#\Diamond^{D}\rceil p = t$.

11) $\#\Diamond^{D}Sp = t$.

12) $\#\Box^{D}(q \leftrightarrow \Box^{D}q) = f$.

13) $\#E^{D}p = t$.

14) $\#\Diamond^{D}p = t$.

15) $\#\Box^{D}\Box^{D}q = f$.

16) $\#[B^{R}x] = t$.

17) $\#[J^{R}x] = t$.

18) $\#[x] = t$.

19) $\#\Box^{D}[x] = t$.

20) $\#[\Box^{R}x] = t$.

21) $\#G^{D}[B^{R}x] = t$. In the interpretation under consideration, the symbol B^{R} stands for the above-tabularly-defined *evaluation-function* "positive *aesthetic value (beauty) of* (what, whom) *x*". See table 1.

22) $\#G^{D}[J^{R}x] = t$. In the interpretation under consideration, the symbol J^{R} stands for the above-tabularly-defined *evaluation-function* "positive *hedonistic value (pleasantness) of* (what, whom) *x*", or in other words, "*happiness with, or joy from* (what, whom) *x*". See table 2.

23) $\#(B^{R}x{=}{+}{=}J^{R}x) = t$.

24) $\#(B^{R}x{=}{+}{=}g) = f$.

25) $\#(B^{R}x{=}{+}{=}b) = f$.

26) $\#\Box^{D}G^{D}[B^{R}x] = f$.

27) $\#\Box^{D}W^{D}[B^{R}x] = f$.

28) $\#G^{D}p = t$.

29) $\#W^{D}p = f$.

30) $\#\Box^{D}\neg p = f$.

In relation to the above-defined *interpretation* # of the formal theory $Φ^{DR}$, the axioms AX*-1 – AX*-11 are true, the definition DF*-1 is true, and the only formal logical derivation rule (*modus ponens*) preserves the truth, consequently, the suggested *interpretation* # of the formal theory $Φ^{DR}$, is a *model* of/for this formal theory and, consequently, $Φ^{DR}$ is logically *consistent* (as there is a *model* of/for it).

2. *A Proof of Unprovability of* $(©^D[x] \leftrightarrow D^D[x])$ *in* $Φ^{DR}$*, and a Proof of Unprovability of* $([©^R x] \leftrightarrow [D^R x])$ *in* $Φ^{DR}$

In realtion to the schemes of theorems $(©^D[x] \leftrightarrow D^D[x])$ and $([©^R x] \leftrightarrow [D^R x])$, the following nontrivial questions arise in $Φ^{DR}$. Is constructiveness (or constructivity) identical to algoritmicalness? Is it necessary that all constructions are algorithmical? Can it be possible in mathematics that an object can be constructed but an algorithm of construction of the object does not exist? Abstractly speaking, in principle, it may happen that one (for example a mathematician, logician, specialist in informatics-and-compurer-science) can disagree with biconditionals $(©^D[x] \leftrightarrow D^D[x])$ and $([©^R x] \leftrightarrow [D^R x])$ as necessarily *universal* statements, and can provide concrete counterarguments grounding the disagreement. For having an adequate attitude to this nontrivial abstract possibility, it is worth making acquaintance with [Мартин-Лёф 1975; Нагорный 2010; Непейвода 2010; 2011; 2012; 2014]. Concerning the above-said of the hypothetic possibility of rational counterargumentation against the equivalences $(©^D[x] \leftrightarrow D^D[x])$ and $([©^R x] \leftrightarrow [D^R x])$, it is relevant to remark here that these equivalences are not provable in $Φ^{DR}$.

To prove their unprovability in $Φ^{DR}$, let us undertake the following steps. At first, let us make up a new formal axiomatic theory £ by adding a new axiom, namely AX*-12: $\neg(©^D[x] \leftrightarrow D^D[x])$, to the above-placed list of axioms of $Φ^{DR}$. A model of/for the new formal axiomatic theory £ is created by adding a couple of new items, namely, the following 31) and 32) to the above-given definition of interpretation-function #. Let “#+31&32” be the name of/for the new function to be used as an interpretation of/for £.

31) $\#©^D[x] = t$.

32) $\#D^D[x] = f$.

In relation to the above-defined *interpretation* # of the formal theory Φ^{DR}, the axioms AX*-1 – AX*-12 are true, the definition DF*-1 is true, and the only formal logical derivation rule (*modus ponens*) preserves the truth, consequently, the suggested *interpretation* "#+31&32" of the formal theory £, is a *model* of/for this formal theory and, consequently, £ is logically *consistent* (as there is a *model* of/for it).

Let us assume that $(©^{D}[x] \leftrightarrow D^{D}[x])$ is formally provable in Φ^{DR}. Then £ is logically inconsistent as both $(©^{D}[x] \leftrightarrow D^{D}[x])$ and $\neg(©^{D}[x] \leftrightarrow D^{D}[x])$ are formally provable in £. But it is proved that £ is logically consistent as it has a model. Consequently, our assumption is wrong, consequently, $(©^{D}[x] \leftrightarrow D^{D}[x])$ is not provable in Φ^{DR}. Here we are.

Now let us create a new formal axiomatic theory Ə by adding a new axiom, namely AX*-12: $\neg([©^{R}x] \leftrightarrow [D^{R}x])$ to the above-placed list of axioms of Φ^{DR}. A model of/for the new formal axiomatic theory Ə is created by adding a couple of new items, namely, the following 33) and 34) to the above-given definition of interpretation-function #. Let "#+33&34" be the name of/for the new function to be used as an interpretation of/for Ə.

33) $\#[©^{R}x] = t$.

34) $\#[D^{R}x] = f$.

In relation to the above-defined *interpretation* # of the formal theory Φ^{DR}, the axioms AX*-1 – AX*-12 are true, the definition DF*-1 is true, and the only formal logical derivation rule (*modus ponens*) preserves the truth, consequently, the suggested *interpretation* "#+33&34" of the formal theory Ə, is a *model* of/for this formal theory and, consequently, Ə is logically *consistent* (as there is a *model* of/for it).

Let us assume that $([©^{R}x] \leftrightarrow [D^{R}x])$ is formally provable in Φ^{DR}. Then Ə is logically inconsistent as both $([©^{R}x] \leftrightarrow [D^{R}x])$ and $\neg([©^{R}x] \leftrightarrow [D^{R}x])$ are formally provable in Ə. But it is proved that Ə is logically consistent as it has a model. Consequently, our assumption is wrong, hence, $([©^{R}x] \leftrightarrow [D^{R}x])$ is not provable in Φ^{DR}. Here we are.

3. *Formal Proofs of* $(A^{D}\alpha \supset (©^{D}[x] \leftrightarrow D^{D}[x]))$ *and* $(A^{D}\alpha \supset ([©^{R}x] \leftrightarrow [D^{R}x]))$ *in* Φ^{DR}

Notwithstanding the above-said, $(A^{D}\alpha \supset (©^{D}[x] \leftrightarrow D^{D}[x]))$ and $(A^{D}\alpha \supset ([©^{R}x] \leftrightarrow [D^{R}x]))$ are formally provable in Φ^{DR}. To examine this statement, readers are invited to make acquaintance with the following formal proofs.

1) $\vdash A^D\alpha \leftrightarrow (K^D\alpha \;\&\; (\neg\Diamond^D\neg\alpha \;\&\; \neg\Diamond^D S\alpha \;\&\; \Box^D(\beta \leftrightarrow \Omega^D\beta)))$: axiom scheme AX-3.
2) $\vdash A^D\alpha \supset (K^D\alpha \;\&\; (\neg\Diamond^D\neg\alpha \;\&\; \neg\Diamond^D S\alpha \;\&\; \Box^D(\beta \leftrightarrow \Omega^D\beta)))$: from 1) by elimination of $\leftrightarrow$.
3) $A^D\alpha$ assumption.
4) $A^D\alpha \vdash (K^D\alpha \;\&\; (\neg\Diamond^D\neg\alpha \;\&\; \neg\Diamond^D S\alpha \;\&\; \Box^D(\beta \leftrightarrow \Omega^D\beta)))$: from 2) and 3) by *modus ponens*.
5) $A^D\alpha \vdash \Box^D(\beta \leftrightarrow \Omega^D\beta)$: from 4) by elimination of &.
6) $\vdash A^D\alpha \supset (\Box^D\beta \supset \beta)$: axiom scheme AX-1.
7) $\vdash A^D\alpha \supset (\Box^D(\beta \leftrightarrow \Omega^D\beta)) \supset (\beta \leftrightarrow \Omega^D\beta))$: from 6) by substituting $(\beta \leftrightarrow \Omega^D\beta)$ for β.
8) $A^D\alpha \vdash (\Box^D(\beta \leftrightarrow \Omega^D\beta)) \supset (\beta \leftrightarrow \Omega^D\beta))$: from 3) and 7) by *modus ponens*.
9) $A^D\alpha \vdash (\beta \leftrightarrow \Omega^D\beta)$: from 5) and 8) by *modus ponens*.
10) $A^D\alpha \vdash (\beta \leftrightarrow \Xi^D\beta)$: from 9) by substituting Ξ^D for Ω^D.
11) $A^D\alpha \vdash (\Xi^D\beta \leftrightarrow \beta)$: from 10) by commutativity of $\leftrightarrow$.
12) $A^D\alpha \vdash (\Xi^D\beta \leftrightarrow \Omega^D\beta)$: from 11) and 9) by transitivity of $\leftrightarrow$.
13) $\vdash (A^D\alpha \supset (\Xi^D\beta \leftrightarrow \Omega^D\beta))$: from 12) by introduction of $\supset$.
14) $\vdash (A^D\alpha \supset (©^D[x] \leftrightarrow D^D[x]))$: from 13) by substituting: $©^D$ for Ξ^D, D^D for Ω^D, and [x] for β.

Here you are: $(A^D\alpha \supset (©^D[x] \leftrightarrow D^D[x]))$ is formally proved in Φ^{DR} by the above-presented succession 1)–14). Now let us continue this succession in the following way.

15) $\vdash A^D\alpha \supset (\Omega^D[t_k] \leftrightarrow [\Omega^R t_k])$: axiom scheme AX-7.
16) $\vdash A^D\alpha \supset (©^D[x] \leftrightarrow [©^R x])$: from 15) by substituting: $©^D$ for Ω^D; $©^R$ for Ω^R; x for t_k.
17) $\vdash A^D\alpha \supset (D^D[x] \leftrightarrow [D^R x])$: from 15) by substituting: D^D for Ω^D; D^R for Ω^R; x for t_k.
18) $A^D\alpha \vdash (©^D[x] \leftrightarrow [©^R x])$: from 16) and 3) by *modus ponens*.
19) $A^D\alpha \vdash (D^D[x] \leftrightarrow [D^R x])$: from 17) and 3) by *modus ponens*.
20) $A^D\alpha \vdash (©^D[x] \leftrightarrow D^D[x]))$: from 12) by substituting: $©^D$ for Ξ^D, D^D for Ω^D, and [x] for β.
21) $A^D\alpha \vdash (©^D[x] \leftrightarrow [D^R x])$: from 20) and 19) by transitivity of $\leftrightarrow$.
22) $A^D\alpha \vdash ([©^R x] \leftrightarrow ©^D[x])$: from 18) by commutativity of $\leftrightarrow$.

23) $A^D\alpha \vdash\!\!- ([©^R x] \leftrightarrow [D^R x])$: from 22) and 21) by transitivity of $\leftrightarrow$.

24) $\vdash (A^D\alpha \supset ([©^R x] \leftrightarrow [D^R x]))$: from 23) by introduction of $\supset$.

Here we are: $(A^D\alpha \supset ([©^R x] \leftrightarrow [D^R x]))$ is formally proved in Φ^{DR} by the above-presented succession 1)–24).

Conclusion

According to the present article, there is a one-to-one correspondence between elements of the set of modalities *de dicto* and elements of the set of modalities *de re*. Moreover, corresponding modalities *de dicto* and *de re* are equivalent, under the precisely defined *extraordinary* condition that knowledge is *a priori* (in exactly that meaning of "*a priori*", which is precisely defined by the axiomatic system Φ^{DR}). Consequently, if in some concrete domain of knowledge, there are grounds for negating equivalence of some corresponding *de dicto* and *de re* modalities, then the concrete domain contains *empirical* knowledge, consequently, in this concrete domain, knowledge is *not pure a priori*. According to the given article, from the one's disagreement with equivalence of constructiveness and algoritmicalness, it follows logically by *modus tollens* that according to the one, generally speaking, it is false that $A^D\alpha$. Hence, even in the concrete interpretation resulting in the one's concrete counterexamples, schemes of formulae $(A^D\alpha \supset (©^D[x] \leftrightarrow D^D[x]))$ and $(A^D\alpha \supset ([©^R x] \leftrightarrow [D^R x]))$ are schemes of true statements. Thus, the extraordinary equivalence of corresponding modalities *de dicto* and *de re* essentially depends on the relevant epistemic context, namely, on accepting or rejecting the *extraordinary* epistemic assumption that knowledge is pure *a priori.*

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